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A cupful of limacons.

Bogacki, Przemyslaw (ed.) et al., Proceedings of the 8th annual international conference on technology in collegiate mathematics, ICTCM 8, Houston, TX, USA, November 16–19, 1995. Norfolk, VA: Old Dominion University, Dept. of Mathematics and Statistics (ISBN 0-201-69558-8). Electronic paper (1995).

Have you ever examined the light patterns on the bottom of a cup or bowl with reflective sides and wondered what shapes you see? If a light source is placed somewhat off center, rather than directly above, you might conjecture that the pattern resembles a cardioid. The purpose of this paper is to discuss how students may furnish some answers by using elementary vector algebra and orthogonal projections with the aid of a computer algebra system (CAS). For instance, if the cup is a simple right circular cylinder, then it will become readily apparent that the image is generated by infinitely many limacons. We suppose the unit disk $x^2+y^2 \leq 1$ is the bottom of the cup and the sides are formed by revolving the graph of a smooth function $x=f(z)$ with $f(0)=1$ and $f'(z) \geq 0$ about the z -axis. For example, $f(z)=1$ when the cup is a cylinder. A CAS such as Mathematica, for example, makes it easy to model the reflected light rays formally, without even specifying exactly what $f(z)$ is. By then making particular choices for $f(z)$, we may use the plotting capabilities a CAS provides to assess the accuracy of the model by comparing the graphs with what is seen in reality. (author's abstract) (the paper is available under <http://archives.math.utk.edu/ICTCM/EP-8.html>)

Classification: G74 R24

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