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Fine, Benjamin; Gaglione, Anthony M.; Rosenberger, Gerhard

Introduction to abstract algebra. From rings, numbers, groups, and fields to polynomials and Galois theory.

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The book under review provides an introduction to the basic concepts, methods, and results of abstract algebra for undergraduate students meeting this crucial component of any mathematics curriculum the first time. Based upon their rich experience teaching the subject at various levels, the authors have chosen an approach that develops the theories of algebraic structures from more familiar and well-studied topics such as the number systems and their algebraic properties, elementary number theory of the integers, geometric vectors and transformations in two and three dimensions, and others. This method is meant to show how the abstract structures arise quite naturally from elementary problems in classical mathematics, thereby allowing beginner-level students to follow the process of abstraction progressively and effectively likewise. The authors themselves describe their leading didactic principle as follows: well-known system to abstract structure to uniqueness of the original system, and this approach is pursued throughout the entire textbook. As for the precise contents, the book comprises fifteen chapters. Each chapter is divided into several sections and subsections, respectively, covering various basic topics of algebra under the following titles of the single chapters. 1. Abstract algebra and algebraic reasoning. – This chapter gives a general introduction to what follows, together with a brief account of integers and the induction principle. 2. Algebraic preliminaries. – Here, some basic naive set theory is discussed, including binary operations, general algebraic structures and their isomorphisms, and a first introduction to groups. 3. Rings and integers. – Starting from the ring of integers, some elementary ring theory is explained, ending with the proof that, up to isomorphism, $\mathbb{Z}$ is the unique ordered integral domain satisfying the principle of mathematical induction. 4. Number theory and unique factorization. – After the basics of elementary number theory, this chapter leads the reader to the idea of a general unique factorization domain. 5. Fields: the rationals, reals and complexes. – The construction of the number systems $\mathbb{Q}$, $\mathbb{R}$ and $\mathbb{C}$ serves here as a motivation for the general concepts of fields and division rings. 6. Basic group theory. – Apart from the fundamental facts on groups, subgroups, group homomorphisms and cyclic groups, the symmetric groups and geometric transformation groups are described as important examples. 7. Factor groups and group isomorphism theorems. 8. Direct products and abelian groups. – This chapter culminates in a proof of the structure theorem for finitely generated abelian groups. 9. Symmetric and alternating groups. – Here, the cycle structure of these types of groups is explained, and the proof of the simplicity of A_n for $n \geq 5$ is given. 10. Group actions and topics in group theory. – The Sylow theorems and some of their applications, the classification of groups of small order, solvable groups, composition series of groups, and the Jordan-Hölder theorem are the highlights in this more advanced chapter. 11. Topics in ring theory. – The topics treated here include a little ideal theory in commutative rings as well as principal ideal domains and their unique factorization property. 12. Polynomials and polynomial rings. – In this chapter, the focus is on polynomial rings over a field and their ring-theoretic properties, including zeros of polynomials, the fundamental theorem of algebra, and the notions of algebraic and transcendental numbers. Also, polynomial rings over integral domains are touched upon, and Gauss's theorem on the unique factorization in $[RX]$ for a UFD-ring R is presented. 13. Algebraic linear algebra. – This part introduces abstract linear algebra over an arbitrary field, with a brief discussion of inner product spaces and the Gram-Schmidt orthogonalization process for real vector spaces. 14. Fields and field extensions. – Algebraic field extensions, splitting fields, algebraic closures, finite fields, and transcendental field extensions are the main objects of study in this chapter, just as its title indicates. 15. A survey of Galois theory. – After a short overview of the idea of Galois theory, this final chapter is devoted to Galois extensions, Galois groups, the fundamental theorem of Galois theory, and some of the classical applications such as the insolvability by radicals of the general polynomial equation of fifth degree and the impossibility of some ruler and compass constructions. Each chapter ends with a generous supply of related exercises of varying degrees of difficulty, where most of the problems come equipped with concrete hints or additional instructions. All together, the present book provides a very lucid and instructive introduction to first-time algebra students. The utmost detailed presentation of the core material, the wealth of illustrating examples, and the many outlooks for further study make this excellent algebra primer a highly welcome, useful and valuable addition to the abundant textbook literature in the field. It should be pointed out that there is a related introductory algebra text by two of the authors (*B. Fine* and *G. Rosenberger* with *C. Carstensen*), with a slightly different flavor [Abstract algebra. Applications to Galois theory, algebraic geometry and cryptography. Berlin: Walter de Gruyter; Lemgo: Heldermann Verlag (2011; Zbl 1213.00004)], but with many overlappings otherwise.

Werner Kleinert (Berlin)

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