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More calculus of a single variable.

Undergraduate Texts in Mathematics. New York, NY: Springer (ISBN 978-1-4939-1925-3/hbk; 978-1-4939-1926-0/ebook). xvi, 411 p. (2014).

This book is concerned with the qualitative and numerical treatment of some advanced aspects of the calculus of a single variable. The volume goes beyond the basics of a first course in calculus to reveal the power and richness of the subject. Several topics from calculus are considered, including real numbers, continuity, differentiation and integration, mean value theorems, the exponential function. At the same time, the author provides various applications, such as the AGM inequality, convexity, integration techniques, and explicit formulas for π. The content of the book is divided into 14 chapters, as follows: 1. The real numbers; 2. Famous inequalities; 3. Continuous functions; 4. Differentiable functions; 5. The mean value theorem; 6. The exponential function; 7. Other mean value theorems; 8. Convex functions and Taylor's theorem; 9. Integration of continuous functions; 10. The fundamental theorem of calculus; 11. Techniques of integration; 12. Classical examples; 13. Simple quadrature rules; 14. Error terms. In what follows, I describe some features of this book. Particular care is paid to three consequences of the intermediate value theorem, which are often overlooked: the universal chord theorem, the average value theorem for sums and its weighted version, the mean value theorem for sums. The latter two are so named because of their integral analogues, the average value theorem and the mean value theorem for integrals. Another feature of this book concerns the definite integral, which is developed as an extension of the notion of the average value of a continuous function evaluated at N points. The definite integral's relationship with area is also discussed. Some important series are studied in this book, such as geometric series, p-series, the alternating harmonic series, the Gregory-Leibniz series, and some Taylor series. However, there is no treatment of power series, tests for convergence, radius of convergence. Motivated by the mean value theorem for the second derivative, error terms are studied in the last chapter of this volume. An inequality can often be recast as an equality which contains an error term. Jensen's inequality, the AGM inequality, Young's integral inequality, and quadrature rules are considered in this way. The book under review is recommended as a text for a course in advanced calculus, as a supplementary text for courses in analysis, and for self-study by students, instructors and applied mathematicians. *Teodora-Liliana Rădulescu (Craiova)*

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