ZMATH 1999f.04362

McDonald, John N.; Weiss, Neil A.

A course in real analysis.

Academic Press, New York, NY (ISBN 0-12-742830-5). 762 p. (1999).

This is a book about real analysis, but it is not an ordinary real analysis book. Written with the student in mind, this text incorporates pedagogical techniques not often found in books at this level. The book is intended for a one-year course in real analysis at the graduate level or the advanced undergraduate level. The text material has been class tested several times and has been used for independent study courses as well. This book contains many features that are unique for a real analysis text. Here are a few. Motivation of key concepts. Detailed theoretical discussion. Illustrative examples. Abundant and varied exercises. Applications. The text offers considerable flexibility in the choice of material to cover. Chapters 1 and 2 (set theory, real numbers, and calculus) present prerequisite material that provides a common ground for all readers. Chapters 3 and 4 present the elements of measure and integration by first discussing the Lebesgue theory on the line (Chapter 3) and then the abstract theory (Chapter 4). This material is prerequisite to all subsequent chapters. Chapter 5 provides an introduction to the fundamentals of probability theory, including the mathematical model for probability, random variables, expectation, and laws of large numbers. In Chapter 6 differentiation is discussed, both of functions and of measures. Topics examined include differentiability, bounded variation, and absolute continuity of fuctions, and a thorough discussion of signed and complex measures, the Radon-Nikodym theorem, decomposition of measures, and measurable transformations. Chapter 7 provides the fundamentals of topological and metric spaces. In addition to topics traditionally found in an introduction to topology, a discussion of weak topologies and function spaces is included. Completeness, compactness, and approximation comprise the topics for Chapter 8. Examined therein are the Baire category theorem, contractions of complete metric spaces, compactness in function and product spaces, and the Stone-Weierstrass theorem. Presented in Chapter 9 are Hilbert spaces and the classical Banach spaces. Among other things, bases and duality in Hilbert space, completeness and duality of $\mathcal{L}^p$-spaces, and duality in spaces of continuous functions are discussed. The basic theory of normed and locally convex spaces is given in Chapter 10. Topics include the Hahn-Banach theorem, linear operators on Banach spaces, fundamental properties of locally convex spaces, and the Krein-Milman theorem. Chapter 11 provides applications of previous chapters to harmonic analysis. We examine the elements of Fourier series and transforms and the $\mathcal{L}^2$-theory of the Fourier transform. In addition, an introduction to wavelets and the wavelet transform is presented. Chapter 12 examines measurable dynamical systems. This chapter requires the one on probability (Chapter 5) and discusses ergodic theorems, isomorphisms of measurable dynamical systems, and entropy.

Classification: I55 I15
Keywords: harmonic analysis; ergodic theorem; entropy; duality; Fourier transform; convex spaces