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The length of spirographic curves.
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If a disk D of radius r rolls, without slipping, on the inside of a larger circle C of radius R, and if P is a fixed point on D at distance $d \leq r$ from its center, then the curve traced by P is called a *hypotrochoid*. As is known to anyone familiar with spirographs, varying R, r, and d results in a rich variety of beautiful curves. Such, and related, curves are discussed by *V. Gutenmacher* and *N. B. Vasilyev* [Lines and curves. A practical geometry handbook. Basel: Birkhäuser (2004; Zbl 1086.51001)]. Notice when $R = 2r$ and $d < r$, one obtains an ellipse. Calling a point on the hypotrochoid with maximal distance from the center of C a *top*, and the arc between two consecutive tops a *bow*, the author of the paper under review proves that if $d < r$, then the length of a bow is equal to $(R-r)/R$ times the perimeter of the ellipse having semi-axes $r+d$ and $r-d$. Amazingly, he does this *à la Euclid*, without any resort to calculus. It is worth mentioning that when $R = 2r$ and $d = r$, then the locus of P turns out, unexpectedly, to be a straight line. This is usually referred to as *Copernicus theorem*, as, for example, on page 3 of the afore-mentioned book of Gutenmacher and Vasilyev, and on page 145 of *H. Steinhaus*'s book [Mathematical snapshots. 3rd Am. ed., rev. and enl. Reprint. Oxford etc.: Oxford University Press (1983; Zbl 0513.00002)]. However, the theorem was known to the Islamic astronomer Naṣīr al-Dīn al-Ṭūsī (1201–1274), who stated and proved it in his *Commentary on the Almagest* (1247) in the context of his solution for the latitudial motion of the inferior planets. Interested readers are referred to *I. N. Veselovsky*'s article [Copernicus and Naṣīr al-Dīn al-Ṭūsī, J. Hist. Astron. 4, No. 2, 128–130 (1973; doi:10.1177/002182867300400205)]. Copernicus theorem is also referred to as *al-Ṭūsī's couple* and *rolling device*. *Mowaffaq Hajja (Irbid)*

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