**ZMATH 2015d.00645**

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**A characterization of the equilateral triangles and some consequences.**

Math. Intell. 36, No. 2, 1-2 (2014).

Equilateral triangles are characterised as having three sides of equal length. The author generalises this characterisation to what he calls Conway's Little Theorem: Equilateral triangles are characterised by the assertion that each ratio of two sides and each ratio of two angles are rational. For the proof, the triangle $ABC$ is placed in the complex plane and scaled such that the side lengths $a$, $b$, $c$ are rational and the angles rational multiples of $\pi$. Expressing the complex number $C$ through $A$, $B$ and the angles and side lengths, Conway finds an equation of the form $c + a\omega^{kq} = b\omega^{kp}$, which holds for all $k$ prime to $n$ (modulo $n$) and where $\omega$ is a primitive $n$th root of unity. This leads to $\phi(n)$ triangles with the given lengths, angles and base $AB$. But there are only two such triangles, which then implies that the angles are positive multiples of $60°$. Hence the triangle is equilateral. Conway also gives two consequences of his Little Theorem: Let a *rational angle* be an angle that is a rational multiple of $\pi$. Then, the only rational angle $\theta$ in the open interval $(0, 90°)$ for which $\cos\theta$ is rational is $\theta = 60°$. The only rational angle $\phi$ in the open interval $(0, 90°)$ for which $\sin\phi$ is rational is $\phi = 30°$. The other consequence is, the only rational angles for which the square of any of the six standard trigonometric functions is rational (or $\infty$) are the multiples of $30°$ and $45°$. *Wolfgang Globke (Adelaide)*

*Classification:* G60
*Keywords:* equilateral triangles
doi:10.1007/s00283-014-9447-3