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The geometry of numbers.

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The geometry of numbers originated with the publication of Minkowski's seminal work in 1896 and ultimately established itself as an important field in its own right. By resetting various problems into geometric contexts, it sometimes allows difficult questions in arithmetic or other areas of mathematics to be answered more easily; inevitably, it lends a larger, richer perspective to the topic under investigation. Its principal focus is the study of lattice points, or points in n-dimensional space with integer coordinates-a subject with an abundance of interesting problems and important applications. Advances in the theory have proved highly significant for modern science and technology, yielding new developments in crystallography, superstring theory, and the design of error-detecting and error-correcting codes by which information is stored, compressed for transmission, and received. This book presents a self-contained introduction to the geometry of numbers, beginning with easily understood questions about lattice points on lines, circles, and inside simple polygons in the plane. Little mathematical expertise is required beyong an acquaintance with those objects and with some basic results in geometry. The reader moves gradually to theorems of Minkowski and others who succeeded him. On the way, he or she will see how this powerful approach gives improved approximations to irrational numbers by rationals, simplifies arguments on ways of representing integers as sums of squares, and provides a natural tool for attacking problems involving dense packings of spheres. An appendix by Peter Lax gives a lovely geometric proof of the fact that the Gaussian integers form a Euclidean domain, characterizing the Gaussian primes, and proving that unique factorization holds there. In the process, he provides yet another glimpse into the power of a geometric approach to number theoretic problems.

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