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Clark, Pete L.; Diepeveen, Niels J.
Absolute convergence in ordered fields.
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Let $\mathbb{F}$ be an ordered field. The authors show that the relation between convergence and absolute convergence of an infinite series $\sum_{n=1}^{\infty} a_n$ in $\mathbb{F}$ is closely connected to the sequential (i.e., Cauchy) completeness of the field $\mathbb{F}$. indent=8mm

(i) If $\mathbb{F}$ is sequentially complete and Archimedean, then $\mathbb{F}$ is isomorphic to $\mathbb{R}$; then, in this case, it is well known (from calculus) that every absolutely convergent series in $\mathbb{F}$ is convergent in $\mathbb{F}$, but the converse is not true, that is, $\mathbb{F}$ has a convergent series that is not absolutely convergent (e.g., let $a_n = (-1)^n/n$).

(ii) If $\mathbb{F}$ is sequentially complete and non-Archimedean, then $\sum_{n=1}^{\infty} a_n$ converges in $\mathbb{F}$ if and only if it converges absolutely in $\mathbb{F}$.

(iii) If $\mathbb{F}$ is not sequentially complete ($\mathbb{F}$ may be Archimedean, that is, isomorphic to a proper subfield of $\mathbb{R}$, or non-Archimedean), then $\mathbb{F}$ has an absolutely convergent series that is not convergent and $\mathbb{F}$ has a convergent series that is not absolutely convergent.

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