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Linear algebra. Step by step.

Oxford: Oxford University Press (ISBN 978-0-19-965444-4/pbk). viii, 608 p. (2014).

Linear algebra plays an important role in many fields where students are not normally strong in mathematics. The author has written this text with these students in mind. To this end he has incorporated a large number of examples and step-by-step explanations of every new concept and result, as well as many numerical and also theoretical exercises with solutions at the back of the book or available online. The students are thus led by the hand and spoon-fed all the way which in the reviewer's opinion makes the book not very suitable for mathematics students. The chapter headings are: 1. Linear equations and matrices, 2. Euclidean space, 3. General vector spaces, 4. Inner product spaces, 5. Linear transformations, 6. Determinants and the inverse matrix, 7. Eigenvalues and eigenvectors. The book is rigorous in the sense that definitions, results and proofs are stated carefully. There are, however, some inconsistencies and shortcomings. Chapter 2 bridges the gap between the vector geometry learnt at school and the concept of an n-dimensional vector space, while Chapter 3 presents the formal definition of vector spaces. In Chapter 2, the concept of the angle (measured in degrees) between two vectors (two-dimensional) suddenly appears from nowhere and the cosine rule is fished out of trigonometry. This is then generalized to n dimensions by means of the formula normally derived from the Cauchy-Schwarz inequality, now using radians which remain unexplained, while the Cauchy-Schwarz inequality is then proved using the above-mentioned formula! In Chapter 3, the Cauchy-Schwarz inequality reappears and is now proved in the usual way, while angles make no appearance. The student will no doubt ask why the earlier simple proof does not work here. An important shortcoming is that the scalars are assumed to be real numbers. Complex numbers are nowhere mentioned. In the last chapter, eigenvectors are introduced as those non-zero vectors that are just scaled by the matrix, the scaling factor being the associated eigenvalue. In the reviewer's opinion a statement like 'It is a relationship like mother and child because eigenvalues give birth to eigenvectors' is not very helpful. On the one hand, the direction wrong, on the other, mothers can have several children and in this way the statement makes sense, but this fact is not mentioned. Then eigenvalues are determined via the characteristic equation of a matrix. Here, the above-mentioned shortcoming comes to bear: nowhere is there a caveat about complex roots. The student is thus led to expect all eigenvalues to turn out real! This leads to other pitfalls. The author defines a matrix to be diagonal if all its off-diagonal entries vanish, thus implicitly assuming that all the diagonal elements are real numbers. Next, the author defines a matrix A to be orthogonally diagonalizable if there is an orthogonal matrix Q such that $Q - 1AQ$ is diagonal. This leads the author to the spectral theorem: A matrix is orthogonally diagonalizable if and only if it is symmetric, which is, of course, correct with the given definitions. But it is misleading because the usual spectral theorem is: A square matrix over $\mathbb{C}$ is unitarily similar to a diagonal matrix if and only if it is normal, with the corollary: A square matrix over $\mathbb{R}$ is orthogonally similar to a diagonal matrix if and only if it is symmetric (cf. the excellent textbooks by Blyth and Robertson). Here the word 'diagonal' is tied to the field considered. It would help the student a great deal if this more general context had been elucidated. The book also contains brief historical biographies of those mathematicians involved in the development of linear algebra, as well as a number of interviews with people actively using linear algebra. It will certainly be welcomed by those who find more formal introductions too daunting. *Rabe von Randow (Bonn)*

Classification: H65
Keywords: textbook; linear equations; Euclidean space; vector space; inner product space; linear transformation; determinant; inverse matrix; eigenvalue; eigenvector