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**Schäffer, Juan Jorge**
**Basic language of mathematics.**
Hackensack, NJ: World Scientific (ISBN 978-981-4596-09-1/hbk; 978-981-4596-11-4/ebook). x, 310 p. (2014).

This well written book explains the underlying structure of sets and number systems in a modern style which could see it replace, say, [*P. R. Halmos*, Naive set theory. York-London: D (1960; Zbl 0087.04403)]. The author, Juan Jorge Schäffer, states that "philosophical elucidation" or a "rigorous account of the foundations of mathematics" are not the aim of this text. The aim is "to clarify the usage of the fundamental concepts, derive their simplest properties and relationships, and make the language they constitute available for use". The chapters are divided into short sections for ease of reading and reference within the text. In brief, they cover sets, mappings, families, relations, ordering of sets, natural numbers, counting systems, finite sets, complete ordered fields, the real numbers and infinite sets. Natural numbers are defined as a counting scheme with 0 and a successor map satisfying three axioms. Addition is defined using the successor map and multiplication is developed as iterated addition. Chapter 12 contrasts various properties of finite, countable, infinitely countable and uncountably infinite sets with cardinality given as a minimal listing of elements. Algebraic structure is developed from the modern standpoint of commutative monoids. Fields and complete ordering lead to a definition of $\mathbb{R}$ as the unique complete ordered field. Uniqueness up to isomorphism is shown and from $\mathbb{R}$ the author shows how to extract $\mathbb{N}$, $\mathbb{Z}$ and $\mathbb{Q}$. In Chapter 16 we are shown the equivalence of the existence of natural and real numbers. The text concludes with an exploration of infinite cardinality, the axiom of choice and variates of this axiom including maximality, minimality and ordering properties. The development of real numbers from set theory is done with fine detail and distinction. For instance, the word function is reserved for $\mathbb{R}$ and $\mathbb{C}$, the word mapping used on sets with less structure. Sets may have a partition but indexed sets are called families and may have a classification. Such detail calls for unambiguous and specific notation. The overall style of that notation is consistent and the author supplies very detailed indexes of terms, names, conditions and symbols to help support the reader's memory. Juan Jorge Schäffer intends the book to be of benefit to graduate students and has used it to support the comprehensive honors program at Carnegie Mellon University. This may account for several unusual aspects such as propositions given without proof, predominately in the early chapters but still present with Proposition 73A, on page 114, about one third way through the book. Perhaps these are meant as exercises as there are no set exercises given for the reader. An understanding of mathematical logic is assumed, or more precisely, the reader is referred to Chapter 1 and part of Chapter 3 of [*A. M. Gleason*, Fundamentals of abstract analysis. New ed. New ed. Boston, MA: Jones and Bartlett Publishers (1991; Zbl 0773.00001)] for concurrent study. Surprisingly, there is no bibliography. *Andrew Percy (Churchill)*

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