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Linear algebra, geometry and transformation.

Textbooks in Mathematics. Boca Raton, FL: CRC Press (ISBN 978-1-4822-9928-1/pbk+ebook; 978-1-4822-9931-1/ebook). xiv, 460 p. (2015).

This textbook comprises an elementary introduction to linear algebra. All the standard topics of a first course are covered, but the treatment omits abstract vector spaces. The focus is solely on $\mathbb{R}^n$, complex numbers do not enter. The solution of systems of linear equations plays a cardinal role. So much is usual. What is unusual is the author's aim to interpret every concept and result geometrically, thus motivating the student to learn to visualize what is going on, rather than just relying on calculations. This is a strong and useful feature. The book sets itself two goals: the spectral theorem for symmetric transformations and the linear rank theorem which plays a key role in the inverse/implicit function theorem for differentiable mappings once they have been linearly approximated. The author's restriction to $\mathbb{R}^n$ leads, however, to certain shortcomings. An eigenvector of a linear transformation T is introduced as a nonzero vector in $\mathbb{R}^n$ that is just scaled by a fixed real factor which is called an eigenvalue of T. The eigenvalues of T are then found to be the roots of the characteristic equation of T. Here, the author then states that this equation has n roots and refers to the fundamental theorem of algebra. At this point it is of cardinal importance to mention complex numbers! For $n = 2$, there need not be any real eigenvalues, e.g. rotations, an example which the author in fact discusses, without however mentioning that the characteristic equation does, of course, have two roots, albeit complex. The author includes the proof of the spectral theorem for symmetric transformations, but makes the following remark: "We regret having to leave the realm of algebra and using a fact about continuity . . . . A purely algebraic proof is possible, but we would need to move beyond real scalars and introduce the complex number system, an even bigger departure from the familiar.", an opinion which the reviewer does not share. The book has very many practice sections with over 500 exercises, most of them numerical. Applications of linear algebra are not included. As the author mentions in the preface, it was his aim to provide a sound mathematical introduction, and in the reviewer's opinion he has succeeded in doing this. *Rabe von Randow (Bonn)*