ZMATH 2015b.00763
Bourchtein, Andrei; Bourchtein, Ludmila
CounterExamples. From elementary calculus to the beginnings of analysis.
Textbooks in Mathematics. Boca Raton, FL: CRC Press (ISBN 978-1-4822-4667-4/hbk). xxix, 331 p. (2015).

Mathematicians solve problems and teach how to solve them. In all cases, examples are very useful to illustrate abstract results and the general framework. Very often in practice, a good understanding of the right example clarifies the meaning and importance of a general statement or even leads to the formulation and proof of a theorem. Beyond that, an example can show that a certain implication is false. This is a "counterexample", which is extremely useful in many cases. Any analyst is always concerned about good counterexamples to use in lectures and research. One of the goals of this book is to provide an outlook of important concepts and theorems in calculus and analysis by using counterexamples. The volume is intended as a one-semester undergraduate introduction to counterexamples in calculus and analysis. More precisely, the book under review is devoted to counterexamples that arise in differential and integral calculus on the real axis or in $\mathbb{R}^2$. The first six chapters are related to single-variable functions, starting with elementary properties of functions (partially studied even in college), passing through limits and continuity to differentiation and integration, and ending with numerical sequences and series. The second part (chapters 7-9) deals with function of two variables, involving limits and continuity, differentiation and integration. The exposition is restricted to the main definitions and theorems of calculus in order to explore different versions of the fundamental concepts and to see what happens a few steps outside of the traditional formulations. The book covers topics concerning the functions of real variables, starting with elementary properties, moving to limits and continuity, and then to differentiation and integration. The many examples presented throughout the book typically start at a very basic level and become more complex during the development of exposition. At the end of each chapter, supplementary exercises of different levels of complexity are provided, the most difficult of them with a hint to the solution. This is a valuable book for students and researchers alike.

<div align="right">*Teodora-Liliana Rădulescu (Craiova)*</div>

Classification: I15
Keywords: continuity; differentiability; integration