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Creating symmetry. The artful mathematics of wallpaper patterns.

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This wonderful book describes mathematical techniques to construct symmetric drawings. The approach is new. It combines geometry, vector spaces, abstract algebra, complex functions and Fourier series. The resulting pictures are beautiful. The only background needed is some calculus (derivatives and integrals). All other mathematics needed is developed in the book. Nevertheless reading the book without knowledge of complex numbers and some deeper algebra might be challenging since introductions are short. The book starts with plane curves defined by complex functions. Their symmetry groups and Fourier series are studied. From Chapter 6 on, functions from the complex plane to itself are used to draw symmetric pictures and not only curves. In Chapter 8, functions for creating freeze ornaments are given. The main idea of the book is developed in Chapter 9. Symmetric pictures in the plane are interpreted as waves satisfying wave equations. Algorithmically a normal photograph is transformed into a wallpaper pattern. At first the algebra and analysis of three-fold symmetry is examined as a special case. In Chapters 15 to 18, the method is generalized to construct any type of wallpaper pattern. All 17 crystallographic groups are constructed. In Chapter 19, the author establishes a concept which he calls color-reversing symmetry. This appears in colored wallpaper images. The next chapters are more algebraic studying point groups and local symmetries in wallpaper patterns. In the rest of the book the methods developed are used for polyhedral symmetry and in the hyperbolic plane.

The book contains exercises in a wide range of difficulty. Some of them are with solutions. The book offers a deeper understanding of symmetry by diving into different mathematical fields and thereby enjoying beautiful images. *Stephan Rosebrock (Karlsruhe)*

Classification: A85 M85 H45
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