**ZMATH 2015f.00784**

**Montesinos, Vicente; Zizler, Peter; Zizler, Václav**

**An introduction to modern analysis.**

Cham: Springer (ISBN 978-3-319-12480-3/hbk; 978-3-319-35549-8/pbk; 978-3-319-12481-0/ebook). xxxi, 863 p. (2015).

This monograph presents a combined introduction to both elementary mathematical analysis and real analysis. When referring to "elementary" mathematical analysis I am thinking of topics like an introduction to the real number system as a complete ordered field, convergent sequences and series of real numbers, continuous and differentiable functions on an interval, the Riemann integral, convex functions, power series and uniform convergence. All this is of course covered in this book; actually, the authors explain both the constructive way of introducing $\mathbb{R}$ by Dedekind cuts and the axiomatic way. But there is much more to this book than just the standard material. Items covered here but seldom found in other textbooks include: the Schröder-Bernstein theorem; Dirichlet's theorem on approximation by rational numbers; the Abel, Dirichlet and Raabe tests for convergent series; Lebesgue's theorem on almost everywhere differentiability of monotone functions; Baire-1 functions; the true meaning of the Taylor formula, i.e., $f(x) = P_n(x) + o(|x - x_0|^n)$ under the sole assumption that $f^{(n)}(x_0)$ exists; the change of variables formula of Riemann integration under minimal hypotheses; a number of nontrivial examples like a continuous, nowhere differentiable function, a differentiable function whose derivative is bounded, yet not Riemann integrable, a $C^\infty$-function whose Taylor series at $x_0$ has radius of convergence zero. Another feature is that the authors describe the topology of $\mathbb{R}$ as such, i.e., define the topological space $\mathbb{R}$, which makes it possible to speak about open and closed subsets of $\mathbb{R}$ (and not only intervals) and compactness at an early stage. Thus the Cantor set is introduced, and the Baire category theorem is used in the context of the real line in one of the first chapters. The rather abstract approach to the real line makes it easy for the authors to introduce metric spaces, since the basic vocabulary has already been presented. Also, the Lebesgue measure and the Lebesgue integral are studied, and there is a detailed discussion of Fourier series, starting from Riemann's localisation theorem and proving the convergence theorems of Fejér, Dirichlet, Dini and Jordan. The final chapter contains a concise introduction to functional analysis including Banach spaces and the "three great theorems", Hilbert space theory and elements of spectral theory. Again, let me highlight some aspects that are not so familiar in the textbook literature: Polish spaces; absolutely continuous functions and the fundamental theorem of calculus in the setting of Lebesgue integration; Auerbach bases for finite-dimensional normed spaces; the Ekeland variational principle; Gâteaux and Fréchet differentiability in Banach spaces; periodic distributions. There is one more chapter, somewhat isolated from the rest of the text, dealing with discrete probability theory. The presentation of the material is very detailed and done with great care on the part of the authors; unfortunately, the publisher has introduced a few typos in the process of copy-editing (e.g., incorrect references). Another interesting feature is the collection of more than 600 exercises at the end of the book. All of these come with detailed hints, some of which are just short of complete solutions. Some exercises are standard for students and some are standard for instructors, but a number of them are intriguing for both audiences. For example, topics like Ramsey's theorem, ordinal numbers or the Henstock-Kurzweil integral are developed in the exercises. In conclusion, this monograph covers a wide range of topics and can be warmly recommended to everyone in need of a text- and reference book in real analysis. *Dirk Werner (Berlin)*

*Classification:* I15
*Keywords:* real analysis; functional analysis; textbook
doi:10.1007/978-3-319-12481-0