
ZMATH 2010f.00757**Lang, Serge****Undergraduate algebra. 3rd ed.**

Undergraduate Texts in Mathematics. New York, NY: Springer (ISBN 0-387-22025-9/hbk). xi, 385 p. (2005).

Over the past fifty years, during his long, productive, versatile, and still lasting mathematical career, Professor Serge Lang presented the international mathematical community with more than thirty textbooks in various fields. This is a unique, utmost impressive record within the mathematical literature, not only with regard to the unparalleled number of published textbooks, but also considering their mathematical width, their didactical mastery, their topicality, and their world-wide popularity. Being a passionate researcher and teacher, Serge Lang has established himself as one of the greatest promoters, messengers, perhaps even missionaries of mathematics in our days. Popularizing contemporary mathematics at any possible level, in all its beauty, and in its disciplinary unity – this has been Professor Lang’s mission and merit for the last half-century. Among his popular textbooks is the title “Undergraduate Algebra”, the first edition of which appeared in (1987; Zbl 0627.00002). The second, significantly enlarged edition of this primer of undergraduate modern algebra was published in (1990; Zbl 0699.00002), and now, after another fifteen years, the third edition of this popular textbook has become available. As it is very typical for Professor Lang’s self-demand and style of publishing, he has tried to both improve and up-date his already well-established text. Having left the text of the second edition all through intact, the author has added to it some additional material, mainly in the later, slightly more advanced chapters, and in order to put students in closer contact with genuine, current research in algebra and number theory. Now as before, the text comes with ten chapters and three appendices covering the following topics: (1) Integers; (2) Groups; (3) Rings; (4) Polynomials; (5) Vector Spaces and Modules; (6) Some Linear Groups; (7) Field and Galois Theory; (8) Finite Fields; (9) The Real and Complex Numbers; (10) Basic Set Theory; (A1) The Natural Numbers; (A2) Construction of the Integers; and (A3) Infinite Sets. The new material added to this third edition is found in Chapters 4 and 6 and it is methodologically (and didactically) highly instructive and valuable. Chapter 4 gives now a totally self-contained algebraic treatment of polynomials, culminating in the discussion of the fairly recent Mason-Stothers Theorem [cf. *W. Stothers*, Polynomial identities and Hauptmoduln, *Q. J. Math.*, Oxf. II. Ser. 32, 349–370 (1981; Zbl 0466.12011) and *R. C. Mason*, Diophantine equations over function fields (London Math. Soc. Lect. Notes Ser. 96, Cambridge University Press) (1984; Zbl 0533.10012)], the simplest known proof of it by *N. Snyder* [Elem. Math. 55, No. 3, 93–94 (2000; Zbl 1031.11012)], and its relations to Diophantine equations. This can be seen as some analogue of the so-called “abc Conjecture”, one of the great contemporary conjectures in arithmetic geometry, which is explained, at the end of this chapter, in as elementary a language as possible. Other related conjectures, like the Hall Conjecture and the Szpiro Conjecture, are touched upon along the same line. The additions to Chapter 6 on special linear groups, which probably presents the most outstanding feature of this undergraduate algebra text, describe various decomposition theorems of the special linear groups $SL_n(\mathbb{R})$ and $SL_n(\mathbb{C})$ in their elementary parts. This puts undergraduate students into a first contact with the algebraic aspects of analysis on groups and Lie algebras. In general, and as another feature of this text, the author has tried, wherever possible, to confront the reader with current problems of mathematical research, at an appropriate level, thereby breaking down the barrier between customary undergraduate teaching and advanced research in algebra and its related areas. Numerous examples and exercises accompany this now already classic primer of modern algebra, which, as usual, reflects the author’s great individuality just as much as his unrivalled didactic mastery and his care for profound mathematical education at any level. It is evident that Professor Lang always aspires to enrapture his readers for mathematics, instead of tormenting them by encyclopedic dullness, and that he knows perfectly well how to reach this sublime goal. The present textbook, in its third and slightly enlarged edition, is just another instance of this notorious fact, among the many others, and it will remain one of the great standard introductions to the subject for beginners. *Werner Kleinert (Berlin)*

Classification: H15 H45 H65*Keywords:* undergraduate textbook; commutative rings and algebras; field theory; linear algebra; group theory; number theory; modern abstract algebra; groups; rings; fields; polynomials; linear groups; sets