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The Euler number. (Eulertalet.)

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Summary: The Euler-number is often defined as the alternating sum of the Betti-numbers, or equivalently as the alternate sum of the number of simplices of a given dimension. The problem is that this requires an explicit triangulation. For purposes of actual computation it is much more efficient (and illuminating) to take a more "functorial" approach, namely the Euler-number which behaves like a cardinal number, but with the difference that we can freely mix dimensions. In the article this is illustrated by a variety of examples, compact real surfaces, projective spaces and Grassmannians, curves and hypersurfaces. We also show how to relate the Euler-number to the types of singularities of a vector field and to prove the Gauss-Bonnets formula, relating the integral of the Gaussian curvature to the Euler-number of the surface.

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