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Making sense of eigenvalue-eigenvector relationships: math majors' linear algebra – geometry connections in a dynamic environment.

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Summary: The present qualitative case study on mathematics majors' visualization of eigenvector-eigenvalue concepts in a dynamic environment highlights the significance of student-generated representations in a theoretical framework drawn from [*A. Sierpinska*, in: On the teaching of linear algebra. Dordrecht: Kluwer Academic Publishers. 209–246 (2000; ME 2016a.00721)] modes of thinking in linear algebra. Such an approach seemed to provide the research participants with mathematical freedom, which resulted in an awareness of the multiple ways that eigenvalue-eigenvector relationships could be visualized in a manner that widened students' repertoire of meta-representational competences in coordination with their preferred modes of visualization. Students' expression of visual fluency in the course of making sense of the eigenvalue problem $Au = \lambda u$ associated with a variety of matrices occurred in different, yet not necessarily hierarchical modes of visualizations that differed from matrix to matrix: (i) synthetic/analytic mode manifested in the process of detecting eigenvectors when the sought eigenvector and the matrix-applied product vector were aligned in the same/opposite directions; (ii) analytic arithmetic mode manifested in the case of singular matrices (in the determination of the zero eigenvalue) and invertible matrices with nonreal eigenvalues; (iii) analytic structural mode, though rarely occurred, manifested in making sense of the trajectory (circle, ellipse, line segment) of the matrix-applied product vector and relating trajectory behavior to matrix type. While the connection between the thinking modes and the concreteness-necessity-generalizability triad was not sharp, math majors still frequently implemented the CNG principles, which proved facilitatory tools in the evolution of students' thinking about the eigenvalue-eigenvector relationships.

Classification: H65
Keywords: undergraduate mathematics education; dynamic geometry software; visualization; representation; connection; linear algebra; eigenvectors; eigenvalues; matrices
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