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Real analysis via sequences and series.

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This textbook is based on the central idea that concepts such as continuity, differentiation and integration are approached via the concepts of sequences and series. For this reason, the backbone of the book is represented by the theory of sequences and series, fact which, in contrast with the most textbooks, makes the treatment of the series much earlier. The contents of the book can be described as follows. Chapter 1, entitled "Introduction", presents some preliminaries on the concepts of set, ordered pair, relation, function, induction, complex number and finite sum. In Chapter 2, entitled "Sequences", the concepts of sequence, monotonic sequence, unbounded sequence, limit of a convergent sequence, the Sandwich theorem, and the Cauchy principle are presented and studied. In particular, the Fibonacci's sequence, the harmonic mean, the arithmetic mean, the geometric mean and the sequential definition of the exponential function are studied. In Chapter 3, entitled "Series", the concept of series is introduced and studied in detail. For the series' convergency, the comparison test, the Cauchy's condensation test, the limit comparison test, the ratio test, the root test, the Kummer-Jensen test, Dirichlet's test and Riemann's theorem on the rearrangement of series are presented. Also, the exponential, sine and cosine functions are introduced as sums of infinite series. Limits of functions are introduced and studied in Chapter 4 through the use of convergent sequences. Based on the concept of limit in Chapter 4, in Chapter 5, entitled "Continuity", the concept of continuity and uniform continuity of a function is introduced and studied. Results related to continuity, such as maximum and minimum-value theorem, intermediate-value theorem, fixed-point theorem are presented. Also, the logarithmic function is introduced and then used to prove Gauss' test for infinite series. In Chapter 6, entitled "Differentiability", the concept of limit in Chapter 4 is used to introduce and study the concept of derivative/differentiation. The chain rule, the inverse function theorem, Rolle's theorem, the mean-value theorem, Cauchy's mean-value theorem, Darboux's theorem for derivative, l'Hopital's rule (continuous and discrete form) are studied. The chapter ends with a short account of the differentiation of power series using differential equations to motivate the discussion. Chapter 7, entitled "The Riemann integral", introduces the concept of integral motivated by the idea of area and based on the upper, lower and Riemann integral sums. Basic properties including the mean-value theorem for integrals, the Cauchy-Schwarz inequality, the fundamental theorem of calculus and integration by parts are presented. In particular, a proof of Wallis' formula, followed by Stirling's formula and a proof that π and e are irrational are provided. An application of Riemann's integral and of the mean-value theorem to numerical integration is given. The chapter ends with two short sections on improper integrals and on the integral test for convergence of series, respectively. Chapter 8, entitled "Taylor polynomials and Taylor series", consists of a short account of Taylor series. Chapter 9, entitled "The fixed-point theorem", firstly deals with the fixed-point theorem on compact real intervals and the fixed-point iteration sequence. Then, it is applied to the Newton's method of approximation for the solution of an equation. The final Chapter 10, entitled "Sequences of functions", deals with sequences of functions and uniform convergence of them or of the infinite series of functions. It is motivated by a problem in differential equations. Properties of infinite series of functions, like the term-by-term integration, term-by-term differentiation, Weierstrass'-M-test, comparison test, d'Alembert's ratio test, Cauchy's root test are also presented. Most of the sections are followed by exercises. The textbook is recommended for a first course in mathematical analysis. *Sorin Gheorghe Gal (Oradea)*

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