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Stillwell, John
Yearning for the impossible. The surprising truths of mathematics.
Wellesley, MA: A K Peters (ISBN 1-56881-254-X/hbk). xiii, 230 p. (2006).

In order to develop the theme in the title, the author follows some threads of the mathematical development going from the Greeks' understanding of the rational and their encounter with the irrational numbers, emergence of complex numbers and their relationships with geometry, where then projective geometry enters, with Pappus and Desargues, who are then tied with laws of algebra. The infinitesimal calculus enters through lengths, areas and volumes with the inevitable computations of π. The Euclidean space becomes too mundane and insufficient to solve the emerging problems and this is when spaces of different curvatures (such as the hyperbolic spaces) become the focus of attention. Perhaps surprisingly, higher dimensions come into play only after the hyperbolic geometry came to existence and most notably through Hamilton's efforts to emulate operations with (two-dimensional) complex numbers into three dimensions, unsuccessfully, which then prompts a consolation prize of discovery of quaternions in 4 dimensions. But these then luckily prove to be rather useful in describing motions of rigid bodies (so much so that nowadays the quaternions are a staple of sophisticated computer animations). Not to forget the numbers and their unique factorizations, the author devotes one chapter to the discovery of ideals. Periodicity is discussed through "impossible" objects and the book ends with the mathematical ultimate – the infinity, and the uncountability of the real numbers. The latter is proved by a rarely used straightforward argument that shows that any countable set of reals is of measure zero (i.e. can be covered by intervals whose total sum is less than any prescribed number). The author points to achievements of Jesuit scientists and uses some of their expositional skills, without explicitly telling the reader about this. The text belongs to the group of texts that popularize mathematics. The author, like a magician, shows off the marvelous mathematical tricks to the mesmerized audience and runs through a good portion of mathematics with these tricks. The audience must be impressed and stunned but this strength of the book becomes also its weakness: While the spectator was enchanted, he nonetheless could not possibly repeat any of the tricks, nor would he necessarily understand them, for he is made a spectator, not a participant. The author has, of course, to make compromises when doing an exposition of deepest mathematical constructs, so we find considerable hand-waiving, undeveloped arguments and sometimes falsities. So one does not exactly see how solving the cubic works or why is it that there are exactly five regular polyhedra in 3-D. In the dubious-statement-department, one finds on p.100 that one needs "advanced methods of logic" to decide which is larger $\sin t/t$ or $\cos t/t$, or on p.201 that "potential infinite is an actual infinite." The claim (on p.195) that Poincaré's conjecture is unsolved perhaps needs updating. It seems to the reviewer that figure 3.26 depicts what is $b + a$ (since the correct caption states that it is "adding a to b"), not $a + b$, and the quantities should be commuted appropriately in figures 3.27 and then 3.29 and 3.30. There is a reference list of some notable classical texts as well as a fairly useful index. The book is good for showing beginning (high school) talented students what a beautiful and exciting subject mathematics is; the expert can take it to the beach and think how he would present the subject in a better way. This is a useful book. *Radoslav M. Dimitrić (Uniontown)*

Classification: E20 A30 A80 M80
Keywords: philosophy of mathematics; history of mathematics (20th century); development of mathematics; mathematical ideas; mathematical notions; discovery and invention in mathematics; mathematics and art; mathematics and visualization.