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The secret life of $1/n$: a journey far beyond the decimal point.

Math. Enthus. 13, No. 3, 189-216 (2016).

Summary: The decimal expansions of the numbers $1/n$ (such as $1/3 = 0.3333\dots$, $1/7 = 0.142857\dots$) are most often viewed as tools for approximating quantities to a desired degree of accuracy. The aim of this exposition is to show how these modest expressions in fact deserve have much more to offer, particularly in the case when the expansions are infinitely long. First we discuss how simply asking about the period (that is, the length of the repeating sequence of digits) of the decimal expansion of $1/n$ naturally leads to more sophisticated ideas from elementary number theory, as well as to unsolved mathematical problems. Then we describe a surprising theorem of *K. Girstmair* [Acta Arith. 67, No. 4, 381–386 (1994; Zbl 0827.11004); Am. Math. Mon. 101, No. 10, 997–1001 (1994; Zbl 0839.11049)] showing that the digits of the decimal expansion of $1/p$, for certain primes p , secretly contain deep facts that have long delighted algebraic number theorists.

Classification: F60 F40

Keywords: unit fractions; decimal expansions; digits; length; period; rational numbers; primitive roots; class numbers; number theory; magic and mystery; divisibility; Euler's theorem; random look; digit frequency; binary quadratic forms; binary expansion; abstract algebra; ideal class group; imaginary quadratic fields
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