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Cannons at sparrows.

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This fascinating and entertaining article begins with an apparently simple problem in convex geometry, due to R. Nandakumar and R. Ramana Rao: given a convex shape, can it be partitioned into N pieces so that all pieces have equal area and perimeter? (This is different, except when $N = 2$, from the "cake and icing problem" of dividing the region into pieces that each have the same area and the same portion of the original body's perimeter.) The $N = 2$ case can be solved easily (if unsportingly) by the use of the Borsuk-Ulam theorem. This paper summarizes further progress on the problem, which involves even more diverse heavy artillery. To even represent the topological space of equal-area partitions requires ideas from the theory of *optimal transport*, and particularly a 1938 theorem of Kantorovich on weighted Voronoi diagrams. Once posed in this way, the problem can be converted into a problem in equivariant algebraic topology via the "configuration space / test map scheme". If there is *no* solution to the original problem, then there is an equivariant map from a certain $(n-1)$-dimensional cell complex to the $(n-2)$-sphere. The existence or nonexistence of this map can be determined using equivariant obstruction theory. This in turn leads, via permutahedra, to a question (solved over a hundred years ago) about prime factors of entries in Pascal's triangle, and the final result of the paper, due to the author and Blagojević: if N is a prime power, the desired dissection exists. The paper is clear and well-written, though I was puzzled to find "equivariant obstruction theory" abbreviated as "EOS" twice. This appears to be a typo: it's "EOT" on two other occasions. *Robert Dawson (Halifax)*

Classification: A85 G95 F65
Keywords: dissection; perimeter; area; Voronoi diagram; configuration space; test map; obstruction; equivariant obstruction theory; permutahedron; Pascal's triangle