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Measure and integral. An introduction for undergraduate students. (Maß und Integral. Eine Einführung für Bachelor-Studenten.)

De Gruyter Studium. Berlin: De Gruyter (ISBN 978-3-11-034814-9/pbk; 978-3-11-035064-7/ebook). x, 172 p. (2015).

The book serves as an introduction to the Lebesgue integration theory, and is based on the author's course delivered at the Technische Universität Dresden. The only prerequisites are elements of real analysis and linear algebra. The approach to the definition of the Lebesgue integral is the standard one: The exposition begins with the basics of measure theory (including the construction of the Lebesgue measure), followed by integration of simple functions and measurable functions. The author covers the usual convergence theorems, parameter-dependent integrals, and the Fubini-Tonelli theorem. The text represents a good departure point for the study of probability theory; the author introduces some basic notions such as random variables and expected values, and most results in the book are formulated for integrals with respect to arbitrary measures. (One exception is the change of variables theorem for integrals with respect to the Lebesgue measure.) Moreover, the author discusses not only real-valued measurable functions, but also measurable mappings between two general measure spaces. Despite this generality, the exposition is clear and easy to follow, and all new concepts that are being introduced are properly motivated. The author proceeds at a quick pace, and the choice of more advanced material seems to be primarily motivated by potential applications. Some classical topics (such as the almost everywhere differentiability of the indefinite integral) are missing, and other (such as the Egorov and Luzin theorems) are delegated to the exercises. Indeed, most of the exercises are theoretical in nature, but some computational ones are also included. On the other hand, the book treats topics which are important for functional analysis or probability theory, such as the convolution, Fourier transform, L^p spaces, Riesz representation theorems, etc. I can warmly recommend the book not only as a textbook for students, but also as a reference book for professional researchers.

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