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Advanced modern algebra. Part 1. 3rd edition.

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The first edition of J. Rotman's comprehensive textbook "Advanced modern algebra" was published in 2002 by Prentice Hall/Pearson Education [Zbl 0997.00001], while the thoroughly revised second edition appeared in 2010 within the renowned AMS textbook series "Graduate Studies in Mathematics" [Zbl 1206.00007]. Now, after another five years, there is a third edition of this excellent graduate algebra text by the devoted teacher and author, Joseph J. Rotman, and again it comes with significant changes and improvements. First of all, this new edition is now divided into two separate volumes called Part 1 and Part 2, mainly for didactic reasons with respect to possible course adoptions. In fact, these organizational changes are to reflect the structure of graduate level algebra courses at the University of Illinois at Urbana-Champaign, USA, with a wealth of additional material incorporated to provide a wide range of options for course design worldwide. The book under review is Part 1 of the new third edition, and it consists of the following two basic courses: Galois theory (Course 1) and Module theory (Course 2). As the author points out in the preface, these two courses serve as joint prerequisites for the forthcoming Part 2, in which more advanced topics in ring theory, group theory, algebraic number theory, homological algebra, representation theory, and algebraic geometry will be presented. As a result of the reorganization of the material, the experienced textbook author J. Rotman has rewritten many sections of the foregoing editions, and thereby actually composed a completely new algebra book. As for the precise contents of the present Part 1, there are 15 chapters divided into the above mentioned two courses. Course I comprises the first seven chapters covering the following topics. Chapter A-1 presents the classical solution formulas for cubic and quartic polynomial equations, while Chapter A-2 recalls some basics from (undergraduate) elementary number theory such as divisibility, Euclidean algorithms, and linear congruences. Chapter A-3 gives an introduction to the theory of commutative rings and their ideals, including maximal ideals and prime ideals, finite fields, irreducibility criteria for polynomials, Euclidean rings, principal ideal domains, and unique factorization domains. Chapter A-4 provides the first steps into group theory, ending with the concept of simple groups, whereas Chapter A-5 treats elementary Galois theory via Galois groups of extension fields, solvability of algebraic equations by radicals, the fundamental theorem of Galois theory, and the relevant aspects of group theory. At the end of this chapter, concrete computations of Galois groups of polynomials are presented. Chapters A-6 and A-7 are just two appendices reviewing basic set theory and equivalence relations, on the one hand, and some linear algebra on the other. Course II starts with Chapter B-1 introducing modules over noncommutative rings and chain conditions for both rings and modules. This chapter concludes with diagrams and exact sequences of module homomorphisms, together with the related technical lemmas. Chapter B-2 is devoted to Zorn's lemma and it various fundamental applications to proofs of existence in algebra, including algebraic closures, transcendence bases and Lüroth's theorem. Chapter B-3 turns to the applications of module theory to group theory and linear algebra, with particular emphasis on the structure theorem of finite abelian groups and canonical forms of matrices. This is complemented by further topics in linear algebra: bilinear forms, inner product spaces, and the classical linear transformation groups. Chapter B-4 introduces categories and functors, with the special focus on module categories. In this context, the reader meets here Galois theory for infinite extensions, free and projective modules, injective modules, divisible abelian groups, tensor products, adjoint isomorphisms, and flat modules. Chapter B-5 is titled "Multilinear algebra" and discusses the following topics: algebras and graded algebras, tensor algebra, exterior algebra, Grassmann algebras, exterior algebra and differential forms, determinantal calculus, and related constructions in module theory. Chapter B-6 deals with more advanced topics in commutative algebra. The first part of this chapter explains the principles of the classical algebraic geometry of affine varieties and their morphisms, including two proofs of Hilbert's Nullstellensatz, while the second part discusses some algorithmic aspects of polynomials and the concept of Gröbner bases. Also Course II ends with two appendices, Chapter B-7 introduces inverse limits and direct limits in module categories, with an outlook to adjoint functors, and Chapter B-8 reviews some general topology, with a brief description of topological groups. Actually, these appendices complement the section on Galois theory for infinite field extensions in the previous Chapter B-4, where the respective notions were already used. As in the foregoing editions, each section of the main text comes with numerous related exercises, and the entire book is interspersed with a wealth of illustrating, very instructive examples. In the preface, the author expresses his hope that this new edition of his standard algebra text presents the material in a more natural way, making it simpler to digest and to use. Certainly, this will remain a matter of taste among students and instructors, and the readers will finally decide about that. Nevertheless, the highly experienced teacher J. Rotman has again presented a new didactic arrangement of the fundamental principles of abstract algebra, which is very original, interesting, functional, inspiring, and student-friendly. No doubt, also this rewritten third edition of the author's classic "Advanced modern algebra" will be among the most popular, useful and appreciated textbooks in the field, and the mathematical community should look forward to the appearance of Part 2 of this comprehensive treatise in the not too far future.

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Classification: H45 H65 H75
Keywords: textbook (algebra); groups; rings; fields; Galois theory; modules; categories and functors; linear and multilinear algebra; affine algebraic geometry; Gröbner bases