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On a recursion formula related to confluent Vandermonde.

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Consider the polynomial $p(s) := \prod_{k=1}^{r}(s-\lambda_k)^{n_k}$ where the $\lambda_1, \dots, \lambda_r$ are distinct and the integers $n_k \geq 1$ with $\sum n_k = n$. The confluent Vandermonde matrix $V(p)$ is defined to be the $n \times n$ matrix $[V_1|V_2|\dots|V_r]$ where V_k is an $n \times n_k$ block whose (i,j)th entry equals $\binom{i-1}{j-1}\lambda_k^{i-j}$ (the case where each $n_k = 1$ is the usual Vandermonde matrix). The matrix $V(p)$ arises in various generalized interpolation problems. The present paper offers a new proof of a recursion formula for $v(p) := \det V(p)$:

$$v(p) = v(q)\prod_{k=1}^{r-1}(\lambda_r - \lambda_k)^{n_k n_r}$$

where $q(s) := p(s)/(s-\lambda_r)^{n_r}$. This leads immediately to a classical identity of L. Schendel, namely, $v(p) = \prod_{i<j}(\lambda_j - \lambda_i)^{n_i n_j}$.

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