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**Summing a curious, slowly convergent series.**

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It is well known that unlike the harmonic series the series  $\sum^{(9)} 1/n$  over all reciprocals of integers that do not contain the digit 9 in their base 10 representation converges. This is likewise true of all series  $\sum^{(X)} 1/n$  in which  $n$  is confined to the set of integers that do not contain the string  $X$  in their base 10 representation. The purpose of the paper is to present an efficient algorithm for computing the value  $\psi_X$  of such a series to an accuracy of 100 digits. (Actually, to compute  $\psi_{42}$  was a challenge put forward by [F. Bornemann, D. Laurie, S. Wagon, J. Waldvogel, The SIAM 100-digit challenge. A study in high-accuracy numerical computing. (Philadelphia), PA: Society for Industrial and Applied Mathematics (SIAM). (2004; Zbl 1060.65002)]). The algorithm uses truncation and extrapolation methods and is linked to the Perron-Frobenius theorem on spectral properties of matrices with positive entries. The authors conjecture that for a sequence of  $n$ -digit strings  $X_n$  without periodic patterns  $\lim_n \psi_{X_n}/10^n = \log 10$  and prove a related conjecture for 1-periodic  $X_n$ .

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*Classification:* I35

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