ZMATH 2009e.00493

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Calculus deconstructed. A second course in first-year calculus.

MAA Textbooks. Washington, DC: The Mathematical Association of America (MAA) (ISBN 978-0-88385-756-4/hbk; 978-0-88385-758-8/solutions manual). xvi, 491 p. (2009).

From the back cover: "[The textbook] is a thorough and mathematically rigorous exposition of single-variable calculus for readers with some previous exposure to calculus techniques but not to methods of proofs. [...] Standard topics and techniques [...] are presented in the context of a coherent logical structure. [...] Numerous examples reinforce both practical and theoretical understanding, and extensive historical notes explore the arguments of the originators of the subject." From the Idiosyncracies in the Preface, pp. xi-xii: "After three centuries of calculus texts, it is probably impossible to offer something truly new. However, here are some of the choices I have made: Level: 'Calculus' and 'analysis' – which for Newton et al. were aspects of the same subject – have in our curriculum become separate countries. I have stationed this book firmly on their common border. [...] Logic: An underlying theme of this book is the way the subject hangs together on the basis of mathematical arguments. The approach is relatively informal [...] Limits: The limit idea at the heart of this treatment is the limit of a sequence, rather than the standard $\varepsilon - \delta$ limit of a function [...] Logarithms and exponentials: [...] I have chosen the ahistorical but more natural route of starting with natural powers to define exponentials and then defining logarithms as their inverses [...] History: I have injected some of the history of various ideas, in some narrative at the start of each chapter and in exercises denoted 'History Notes', which work through specific arguments from the initiators of the subject; [...] much of the discussion is based on secondary sources, although where I have had easy access to the originals I have tried to consult them as well." List of chapters (and appendices): 1. Precalculus, 2. Sequences and their Limits, 3. Continuity, 4. Differentiation, 5. Integration, 6. Power Series, A. The Rhetoric of mathematics (Methods of Proof), B. Answers to Selected Problems. The theory presented in each of 42 sections is preceded by very detailed elaboration of examples, with calculations provided with a great care of clearness, also when they are quite elementary ones. To each but the first section there are assigned numerous exercises, altogether nearly 600 items, many of them subdivided into dozens of examples. Again from the Preface, p. xiii, l. 11-18 from above: "The exercises come in four flavors: Practice problems are meant as drill in techniques and intuitive exploration of ideas. Theory problems generally involve some proofs. [...] Challenge problems require more ingenuity and persistence. [...] History notes are hybrids of exposition and exercise, designed to aid an active exploration of specific arguments from some of the originators of the field." [The author announces that there exists a separate Solution Manual accompanying the book.] Historical notes are found in the problem parts of the following sections [Section — Note(s)]: 2.4. Finding limits — An exercise taken from Euclid's *Elements*; Leibniz sums a geometric series. 2.6. Bounded sets — Bolzano's Lemma. 3.1. Continuous functions — on definitions of the trigonometric functions (originated by Euler). 3.2. The Intermediate Value Theorem and inverse functions — Bolzano's proof of the IVT; Cauchy's proof of the IVT. 3.4. Limits of functions — Newton on $\sin\theta/\theta$. 4.2 Formal differentiation — Law of Cosines and Pythagoras' Theorem; Newton's proof of Product Rule; Newton differentiates $\sin x$. 4.7. Extrema revisited — Fermat on maxima and minima. 4.9. Mean value theorems — Cauchy's Mean Value Inequality. 4. 11. Continuity and derivatives (optional) — Bolzano's example. r45Area and the definition of the integral — Areas of circles in Euclid; Areas of circles in Archimedes. 5.3. The Fundamental Theorem of Calculus — Early calculations of $\int x^k dx$; $\int x^k dx, k$ a positive integer; Fermat's alternate calculation of $\int x^k dx, k > 0$ rational; Wallis' calculation of $\int x^k dx, k > 0$ rational; Leibniz's version of the Fundamental Theorem of Calculus; Newton's version of the Fundamental Theorem of Calculus. 5.4. Formal integration I: Manipulation formulas — Leibniz's derivation of integration by parts. 5.7. Improper integrals — Fermat's calculation of $\int x^{-k} dx$. 5.9. Riemann's characterization of integrable functions (optional) — Riemann's discontinuous integrable function. 6.1. Local approximation of functions by polynomials — Taylor's derivation of his series. 6.5. Handling power series — Newton's binomial series; Wallis' representation of π; Leibniz' series; Abel's Theorem; Some series for $\ln 2$; Mercator's series; Euler on e. The Bibliography consists of 57 items, most of them refer to historical original papers or historical surveying books. The items named in the Preface as the main sources of historical informations contained in the book are: *O. Toeplitz*, The calculus. A genetic approach. Chicago, Ill.: The University of Chicago Press (1963; Zbl 0125.00109) and *C. H. Edwards*, The historical development of the calculus. New York - Heidelberg - Berlin: Springer-Verlag (1979; Zbl 0425.01001). *Bogdan A. Choczewski (Kraków)*

Classification: I15 I45 I55 I65

Keywords: real functions; calculus; historical comments; exercises and problems; textbooks