ZMATH 2010b.00611

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Random models and probabilistic physics. (Modèles aléatoires et physique probabiliste.)

Paris: Springer (ISBN 978-2-287-99307-7/pbk; 978-2-287-99308-4/ebook). 572 p. (2009).

Description de l'éditeur: À partir de connaissances élémentaires sur les probabilités, cet ouvrage propose un cours approfondi de physique mathématique stochastique. D'une part, il expose l'aspect classique des applications probabilistes aux sciences physiques, et introduit les principales notions dans un langage clair et compréhensible par tous ; d'autre part – et c'est là sans doute sa grande originalité – il traite de l'aspect quantique des probabilités, qui sont à la base de développements plus récents en physique statistique et en théorie des champs. Il ne néglige pas pour autant les techniques de simulation aléatoire qui intéresseront aussi bien les milieux de la recherche que de l'industrie. Ce livre s'appuie sur la longue expérience d'enseignement de l'auteur auprès d'étudiants en master et de futurs ingénieurs. C'est à eux que l'ouvrage s'adresse en priorité, ainsi qu'aux élèves des classes préparatoires intéressés par les méthodes stochastiques. Des exercices corrigés complètent chaque chapitre et permettent une meilleure compréhension de leur contenu. Une importante bibliographie termine l'ouvrage, laissant au lecteur le loisir d'approfondir quelques-uns des plus beaux thèmes de ce vaste territoire aléatoire, qui est au cœur des préoccupations scientifiques d'aujourd'hui.

Based on the elementary knowledge of the behavior of probabilities, the author, scientific coworker of the "Commissariat á l'Énergie Atomique" (CEA), presents a thoroughgoing, exhaustive course of mathematical stochastics. On the one hand, the basic notations and principles of stochastics are introduced and classical aspects of the application of random models in physics are explained. On the other hand, also quantum-physical aspects are treated, which only recently have been discussed in statistical physics and quantum field theory. Techniques of random simulations applied in scientific research works and in industry are presented. After a short introduction, in the second chapter of the book basic concepts of the probabilistic theory are introduced. A lot of mathematical laws, more than twenty, are presented. The chapter ends with the introduction of the notion of conditional probability. Martingales, especially their decomposition and convergence, are treated in chapter 3. Markov chains, that means processes without memory, are considered in chapter 4. The term "entropy" and ergodic applications are introduced in chapter 5. Chapter 6 on statistical thermodynamics deals with partition functions and the perfect gas model, it considers gases of fermions and bosons as well as the black-body radiation. Critical phenomena, including the Ising model and the mean field theory for spins, and also chaotic systems, are presented in chapter 7. Stochastic simulations and algorithms are introduced in chapter 8. Here, especially Monte Carlo integration, particle transport processes, simulations of Markov chains and stochastic optimisation are explained. Important random processes, with the exclusion of Brownian motion which is considered in detail in chapter 11, are discussed in chapter 9. Markov, punctiform ("processus de comptage"), Poisson, Lévy and second order processes are dealt with especially. Queuing theory is treated in chapter 10. Such queues are classified by the time τ and the duration σ of the interaction between a "client" and "servers", the number s of the servers interacting with the client, the magnitude m of the interaction region, and the kind of interaction, that means by the quadruple $\tau/\sigma/s/m$. For the laws of interaction, the letters G, M, D are chosen for general and independent, Markov, and deterministic physical processes, respectively. The author considers the queues G/G/1, G/M/1, M/G/1, and M/M/s. In the context of the stochastic integrals treated in chapter 12, the author employs the notations introduced for the processes of Wiener and of Brownian motion. He concentrates on the forms of integrals introduced by Itô and Stratonovich, the Girsanov theorem, and on structure equations. Chapter 13 considers stochastic differential equations, the Langevin equation, the solution of the Ito equation, Feller and diffusion processes, white and colored noise, the Feynman-Kac formula and the Fokker-Planck equation. Numerical schemes of Euler, Milstein, Heun, Runge-Kutta and Platen to solve stochastic differential equations are presented in chapter 14. Besides, here the problem of stochastic stability is explained and Lyapunov exponents are considered. Chapter 15 deals with equations containing partial derivatives, that means with elliptic and parabolic equations, the Korteweg-de Vries equation and the Burgers equation. Stochastic oscillations are discussed in chapter 16. Observing physical processes with much noise, it is often useful to return to an original process or process without noise. Thus, the filter principle, the Kalman-Bucy filter and the auto-regressive moving-average (ARMA) process are described in chapter 17. Chapter 18 presents a stochastic method of analysis applicable for infinite dimensional Hilbert (Gaussian) spaces. Some results which have found numerous applications like the stochastic Malliavin derivative, the Clark-Ocone formula and the Skorohod integral, are presented. In quantum mechanics, the state space is an imaginary one. Quantum-physical, or non-commutative, stochastic methods are introduced in chapter 19. Here, the author especially concentrates on the introduction of the master equation and the Caldeira-Leggett equation describing the coupling of a system with a heat reservoir consisting of harmonic oscillators. Free

probabilities, a special case of non-commutative probabilities first described by D. Voiculescu in 1983, are treated in chapter 20. Chapter 20 ends with an introduction into the free stochastic calculus considering the example of free Brownian motion. This free stochastic calculus has found already various applications, above all to determine eigenvalues of random matrices, e.g. in the case of Coulomb systems. The present text is based on the longterm teaching experience of the author in master classes and engineering courses. Thus, it is mainly written as textbook for such students. But it is also of interest for any reader applying stochastic methods. Every chapter contains exercises to intensify the newly obtained knowledge at the end. The work ends with a long list of well chosen references. *Claudia-Veronika Meister (Darmstadt)*

Classification: M50 K50 K60 K90
Keywords: random models; probabilistic physics; quantum-physical stochastic methods; stochastic simulations; Markov processes; Wiener processes; Brownian motion: Ito integral; Stratonovich integral; Lyapunov exponents; Burgers equations; stochastic oscillations; Korteweg-de Vries equation; filter principle; Kalman-Bucy filter; auto-regressive moving-average process (ARMA process); Malliavin derivative; Clark-Ocone formula; Skohorod integral; master equation; Caldeira-Leggett equation; free probabilities: non-commutative probabilities; free stochastic calculation; free Brownian motion
doi:10.1007/978-2-287-99308-4