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How does one cut a triangle? With forewords by Philip L. Engel, Paul Erdős, Branko Grünbaum, Peter D. Johnson jun. and Cecil Rousseau. 2nd ed.
New York, NY: Springer (ISBN 978-0-387-74650-0/pbk; 978-0-387-74652-4/ebook). xxx, 174 p. (2009).

In the second edition of an engagingly written book [see Zbl 0691.52001 , ME 1991h.02126 for the first edition], addressed to bright high school students and undergraduates, whose contributions are very nicely incorporated into the narrative, the author presents problems belonging to discrete and combinatorial geometry. It starts by solving the problem of (1) finding all positive integers n for which every triangle can be cut into n triangles similar to each other (all $n \neq 2, 3, 5$), and the problem of (2) finding all positive integers n for which every triangle can be cut into n triangles congruent to each other (all $n = k^2$, for some $k \geq 1$). It then moves on to provide two proofs for the fact that (3) of any five points in a triangle of unit area there are three that form a triangle of area $\leq \frac{1}{4}$. R. Peng showed in 1989, while a senior high school student, that $\leq \frac{1}{4}$ can be replaced in (3) with $\leq (\sqrt{2} - 1)^2$, and that this is the best possible constant. With $S_\alpha(F)$ defined as the minimal positive integer n such that among any n points located inside or on the boundary of the figure F, there are three points which form a triangle of area at most $\alpha \cdot |F|$ where $|F|$ denotes the area of F, the author shows that $S_{\frac{1}{4}}(F)$ must be 5 or 6 for all convex figures F, and that both 5 and 6 occur (5 whenever F is a triangle, and 6 when F is a regular pentagon). Among the many Erdős conjectures to be found in this book, we mention two: (4) That, among all triangles formed by three of the vertices of any convex pentagon of unit area, one must have an area of at most $\frac{5-\sqrt{5}}{10}$, and that this bound (attained in the case of the regular pentagon) is best possible; (5) Find the smallest and the largest value of α such that for every convex figure F, $S_\alpha(F)$ is 5 or 6. A second part was added to this second edition, on developments since the publication of the first edition (1990). It contains another proof of (1), several results by M. Laczkovicz on cutting triangles, the result of the collaboration of the author with J. H. Conway, regarding the number of unit equilateral triangles it takes to cover an equilateral triangle of side $n+\varepsilon$ (n^2+2 unit equilateral triangles suffice, and it is conjectured that $n^2 + 1$ do not suffice). Somewhat confusingly, it also contains M. Kahle's improvement of $\leq \frac{1}{4}$ in (3) to $\leq \frac{6}{25}$ and his conjecture that the best possible constant in (3) is $\frac{1}{6}$ (although the matter had been settled by Peng's result, and the conjecture disproved). However, Kahle's involvement with the problem fits nicely with the overall message of the book, that elementary mathematics fascinates those who have encountered it while they were in high school long after their Olympiad experiences are over. Kahle had encountered (3) on one of his five attendances of the Colorado Mathematical Olympiad (called into life and run for 26 years by the author) in 1988, and provided the improved bound 20 years later – while a Postdoctoral Fellow at Stanford. The fact that Kahle, a two time gold medalist of the Colorado Mathematical Olympiad, had not been admitted to the University of Colorado at Colorado Springs for grade point average reasons speaks eloquently about both the ineptitude of the admission rules and regulations and the irrelevance of the institution of high school mathematics in the United States of America (which is, at best, concerned with questions of the kind "prove that A implies B using the Pythagorean theorem" (Preface, xxix)), and leaves graduates "without the foggiest idea of what mathematics is" (Preface, xxviii), as the latter assigned Kahle a grade of C in geometry. *Victor V. Pambuccian (Phoenix)*

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