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Dudley, Underwood

A guide to elementary number theory.

The Dolciani Mathematical Expositions 41; MAA Guides 5. Washington, DC: The Mathematical Association of America (MAA) (ISBN 978-0-88385-347-4/hbk). x, 139 p. (2009).

This book offers, on 136 pages (more than 20 of them are actually white), a compact introduction to elementary number theory. It is meant to allow readers to find the standard results "quickly and with no nonsense". Consequently, there are no references to the literature or exercises. The content of the first 20 Chapters covers material that is more or less standard, with quite a few excursions into the realm of recreational mathematics. The second half of the book does not convince this reviewer. Chapter 23 on sums of three squares, which consists of 1 page only, gives the simple result that sums of three squares do not have the form $4^a(8k+7)$, together with the remark that the proof of the converse by Gauss "involves ternary quadratic forms", which is not very surprising given that $x^2+y^2+z^2$ *is* a ternary quadratic form. Chapters 26 and 27 present the Pell equation $x^2-Ny^2=1$ and continued fractions, but the proof that the Pell equation is solvable if N is a positive nonsquare is omitted; this is all the more regrettable since the following chapter on "multigrades" is completely superfluous. The theorem in Chapter 36 on the abc conjecture claims: "If a and b are relatively prime, $a+b=c$, and $c<(\prod_{p|abc}p)^2$ for all a, b, and c, then $x^n+y^n=z^n$ has no solution in positive integers for $n\geq 3$." This result would have deserved a clearer formulation. In addition, the assumption used in this theorem is not a "weak version of the abc conjecture" as claimed, since the statement of the latter involves the expression $k(\varepsilon)(\prod_{p|abc}p)^{1+\varepsilon}$, and it is (of course) not known whether we can take $k(1)=1$. In Chapter 37, Fermat's method of factoring is explained without mentioning Fermat, who then gets credit for Euler's result that a number can be factored if it can be written as a sum of two squares in two essentially different ways. A version of Pollard's rho method is presented, but neither do we find an explanation why it works nor is there any remark on the complexity of this algorithm, so the reader is left wondering why the rho method is any better than picking a number a at random and testing whether $\gcd(a,N)>1$. In Chapter 39 it is claimed that Goldbach made his conjecture in connection with Euler's efforts in proving the Four Squares Theorem; this is nonsense after all. The main weakness of this Guide, however, is that it does not guide the reader at all. The fundamental theorem of arithmetic, according to the author, is "so natural as to not be worth commenting on", but that it is proved nevertheless, "if only for completeness' sake". The importance of the quadratic reciprocity law apparently lies with the fact that it allows us to evaluate (a/p) "if we ever need to do it". The gap between Fermat's Little Theorem and its generalization by Euler is explained by the remark that "mathematical talent was thin on the ground in those days". Readers looking for guidance are better served with Davenport's "Higher Arithmetic" [*H. Davenport*, The higher arithmetic. An introduction to the theory of numbers. Cambridge: Cambridge University Press (2008; Zbl 1204.11002)], which is comparable to Dudley's book in size, but does contain exercises, intelligent comments, references to the literature, and suggestions for further reading.

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Classification: F60

Keywords: Euclidean algorithm; unique factorization; congruences; Chinese Remainder Theorem; primitive roots; Fermat's Theorem; perfect numbers; quadratic residues; quadratic reciprocity law; Pythagorean triples; Fermat's Last Theorem; sums of two squares; sums of four squares; Pell equation; continued fractions; prime number theorem; abc conjecture; transcendental numbers