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Relationship between the volumes of a conical frustrum and a square frustrum.

Rogerson, Alan (ed.), The mathematics education for the future project. Proceedings of the 13th international conference 'Mathematics education in a connected world', Catania, Sicily, Italy, September 16–21, 2015. Münster: WTM-Verlag (ISBN 978-3-942197-44-1/pbk; 978-3-942197-86-1/ebook). Conference Proceedings in Mathematics Education 1, 279-288 (2015).

Summary: The focus of this work is to establish a relationship between the volume of a conical (circular-based) frustrum and a square frustrum. It was also noted in an earlier paper that apart from the records indicating that the Egyptians used a formular which later metamophorsised into a process of dividing the pyramid into two portions first, calculating the areas as A_1 and A_2, and then obtaining the volume of the pyramid as one-third the height multiplied by the sum of the two different areas A_1 and A_2 added to the square-root of the product of the two areas {i.e. $V = \frac{1}{3}h[A_1 + A_2 + \sqrt{A_1A_2}]$}; the usual method of estimating the volume of such dissected solid figures was by first calculating the volume of the original big pyramidal container, chop off the top part as needed, calculate the volume of the chopped-off pyramid, and then subtract the chopped-off volume from the big pyramid. It had earlier been noted in [*M. F. Macrae* et al., New general mathematics for senior secondary schools: student's book 1. 3rd edition. Harlow: Pearson Education (2001), pp.160–173] that the volume of a conical frustrum here designated as V_{CF} is given as $V_{\mathrm{CF}} = \frac{1}{3}\pi h(R^2 + Rr + r^2)$. Where V_{CF} = volume of square-based frustrum, R = radius of the large circular base, r = radius of the small circular top, and h = the height of the conical frustrum. In the process of developing a less-cumbersome model for the volume of a square-based frustrum, the author ["Volume of a square-based frustrum: alternative formula (Lagsamolu equation)", in: Meeting the challenges in science education. Ondo: Babson Press. 81–93 (2011)] considered models for complete pyramids, including that of Circular frustrum, to arrive at a proven formula for a square frustrum as $V_{\mathrm{SF}} = \frac{1}{6}h(D^2+Dd+d^2)$. Where V_{SF} = volume of square frustrum, D = diagonal of the large square base, d = diagonal of the small square top, and h = the height of the square frustrum. An extension of this was to establish a relationship between the volumes of the two. i.e. volume of square frustrum and that of conical frustrum. The consideration process confirmed the existence of a relationship such that if the top and base diagonals of the square frustrum respectively have equal lengths with the top and base diameters of the conical frustrum, the volume of one may be estimated with the knowledge of the other. This was illustrated.

Classification: G30 G40
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