ZMATH 2016d.00682
Körtesi, Péter
The symmetry of the parabola order three and four.
Billich, Martin (ed.), Mathematica V. Proceedings of the Polish-Czech-Slovak mathematical conference, Catholic University Ružomberok, Spišská Kapitula, Slovakia, June 3–5, 2015. Ružomberok: Verbum, Catholic University in Ružomberok Press (ISBN 978-80-561-0296-1/pbk). Scientific Issues, 51-55 (2015).

Summary: The second order polynomial function $f_2 : \mathbb{R} \to \mathbb{R}$, $f_2(x) = ax^2 + bx + c$, shortly the second order function has got its associated graph a parabola (order 2). The symmetry properties of this function are known as well by most of the secondary school students as well. This function has got as symmetry axis the line: $x = -\frac{b}{2a}$, in other words: $f(-\frac{b}{2a} - x) = f(-\frac{b}{2a} + x)$, is satisfied for all $x \in \mathbb{R}$. This symmetry is used to establish the relation between the coefficients and the position of the roots of the equation order 2, related to one or two fixed numbers. The naturally arising question is if the polynomial function order 3 and 4 have got any symmetry or not? We will see in the sequel that the function $f_3 : \mathbb{R} \to \mathbb{R}$, $f_3(x) = ax^3 + bx^2 + cx + d$ has a symmetry point, we will denote it by $S(x_s, y_s)$, for which the relation: $f_3(x_s) - f(x_s - x) = f(x_s + x) - f(x_s)$ holds for all $x \in \mathbb{R}$. Similarly, we will show that the function $f_4 : \mathbb{R} \to \mathbb{R}$, $f_4(x) = ax^4 + bx^3 + cx^2 + dx + e$ has a kind of skew symmetry, which can be visualised due to the so called inflection line.

Classification: G70 I20
Keywords: symmetry of polynomials; pointwise symmetry; axial symmetry