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A concise introduction to analysis.

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The goal of this book is to establish the principles of mathematical analysis both for functions of real variables and for functions of a complex variable in the shortest possible way. The book contains only definitions and theorems that will be used in further chapters, many of them focusing on the final chapter where the prime number theorem is proved. A great deal of effort has been made in order to achieve a self contained brief exposition. The book can be read and understood without resorting to any external reference since the author provides detailed proofs of all results. However, it requires some background in mathematics, not just in basic calculus. In the first chapter, there is a concise yet comprehensive exposition about convex functions and their differentiability properties, exponential and logarithmic functions and the properties of differentiable functions with applications to numerical series and asymptotic limits. At the end of the chapter, the reader finds a topic unusual in this kind of book: infinite products of numbers, whose properties are needed in the ending section. After introducing the Riemann integral on the real line, the author makes some interesting and original considerations about the rate of convergence of Riemann sums. When the integral of a function is approximated by these sums, the corresponding error is estimated by a constant times the uniform norm of the derivatives of the function and in the periodic case, these constants are the Bernoulli numbers. These numbers appear also in the computation of the Riemann zeta function for even integers, as an application of Fourier series. The chapter about the differentiation of functions of several real variables, although starts with an unusual definition of differentiable function, ends with a very useful and self contained presentation of ordinary differential equations that very likely the reader will appreciate. Some interesting applications of the theory of the Riemann integral in several variables are presented, for example to the relationship between the β and Γ Euler functions and to the Stirling formula for the Γ function. Since the theorem on the change of variables for the Riemann integral is omitted, some effort has to be carried out to obtain the formulae for integration in polar, cylindrical and spherical coordinates. Also, an application to Newton's law of gravitation is discussed. Concerning classical theorems, the divergence theorem for star-shaped domains in the plane is proved aiming to a brief introduction to the local theory of analytic functions which is provided in the last chapter. This introduction, despite being short, allows for some applications concerning Bernoulli numbers, the decomposition of rational functions or the calculus of definite integrals. The last chapter contains the proof of the prime number theorem due to D. J. Newmann and D. Zagier. It is not usual to find a proof of this theorem in a text of an introductory character. The proof is not in any way elementary and requires using much of the results of the previous chapters. It is a happy end for the book. It is worth mentioning that in each chapter one can find several examples of the theory discussed as well as a list of working exercises that will be useful to the reader. *Julià Cufí (Bellaterra)*

Classification: I15
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