

ZMATH 2016d.00781**Schmuland, Byron****Linear recurrences via probability.**

Am. Math. Mon. 122, No. 4, 386-389 (2015).

D. Borwein et al. [Am. Math. Mon. 121, No. 6, 486–498 (2014; Zbl 1303.39002)] have presented various approaches to studying the convergence of the recurrently defined sequence

$$x_n = \alpha_m x_{n-1} + \alpha_{m-1} x_{n-2} + \cdots + \alpha_1 x_{n-m}, \quad n > m, \quad (1)$$

with initial conditions $x_n = a_n$, $n = 1, \dots, m$. The author of the paper under review offers two more approaches based on probability theory provided the α_j are positive. Namely, Markov chain techniques allow one to represent the limit L of (x_n) as $L = \sum_{k=1}^m \alpha_k \pi_k$, with π the stationary distribution of the chain that is canonically associated to (1). Another approach makes use of renewal theory. I completely agree with the author's conclusion that "this delightful problem could also enliven an intermediate probability class."
Dirk Werner (Berlin)

Classification: I75 K65*Keywords:* linear mean recurrences; convergence; Markov chain; renewal theorem

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