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Building generalized inverses of matrices using only row and column operations.

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Summary: Most students complete their first and only course in linear algebra with the understanding that a real, square matrix A has an inverse if and only if $\mathrm{rref}(A)$, the reduced row echelon form of A, is the identity matrix I_n. That is, if they apply elementary row operations via the Gauss-Jordan algorithm to the partitioned matrix $[A|I_n]$ to obtain $[\mathrm{rref}(A)|P]$, then the matrix A is invertible exactly when $\mathrm{rref}(A) = I_n$, in which case, $P = A^{-1}$. Many students must wonder what happens when A is not invertible, and what information P conveys in that case. That question is, however, seldom answered in a first course. We show that investigating that question emphasizes the close relationships between matrix multiplication, elementary row operations, linear systems, and the four fundamental spaces associated with a matrix. More important, answering that question provides an opportunity to show students how mathematicians extend results by relaxing hypotheses and then exploring the strengths and limitations of the resulting generalization, and how the first relaxation found is often not the best relaxation to be found. Along the way, we introduce students to the basic properties of generalized inverses. Finally, our approach should fit within the time and topic constraints of a first course in linear algebra.

Classification: H65 N35
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