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Winkler, Christopher K.
A method to find the sums of polynomial functions at positive integer values.
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From the text: When learning the intuition behind definite integration, calculus students often learn how to find the area under a curve by using a Riemann sum. Often, when attempting to find the area under polynomial curves by this method, students are limited by how many formulas they know for the sums of monomials of a positive integer degree n. The most commonly known formula of this variety is for the sum of monomials of degree 1, namely $\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$. Less common are the formulas for the sum of monomials of degree 2 and 3, namely $\sum_{i=0}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ and $\sum_{i=0}^{n} i^3 = \frac{n^2(n+1)^2}{4}$. Most calculus students are taught to memorize these formulas, and are thus not able to find the sums of polynomials of degrees higher than 3. Further research into the area yields Faulhaber's formula, which involves more complex concepts such as the Bernoulli numbers, with which students are often unfamiliar. In this paper, I show a method for deriving these summations for polynomials of higher degrees without using these complex concepts.

Classification: H20 I20 I30 I50
Keywords: polynomial functions; polynomial representations; sums of monomials; positive integers; polynomials of higher degrees; recursion; simultaneous linear equations; definite integrals
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