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Treeby, David; Tang, Jenny
Combinatorial derivations of familiar identities.
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From the text: Finding two ways to enumerate the same collection of objects can often give rise to useful formulae. For instance, the sum $1 + 2 + \cdots + n$ can be interpreted as the maximum number of different handshakes between $n + 1$ people. The first person may shake hands with n other people. The next person may shake hands with $n - 1$ other people, not counting the first person again. Continuing like this gives the above sum. Another approach is to simply realise that each of the $n + 1$ guests shakes hands with n other guests. However, this counts handshakes twice. Therefore, $1 + 2 + \cdots + n = \frac{n(n+1)}{2}$. This article concerns a combinatorial argument that gives rise to the familiar formula for the sum of the first n squares, $1^2 + 2^2 + \cdots + n^2$. For our derivation of the formula we enumerate the same collection of objects two different ways and then equate the results.

Classification: K20 I30
Keywords: combinatorics; enumeration problems; set of consecutive integers; enumerating rising sequences; sums of squares
https://www.parabola.unsw.edu.au/files/articles/2010-2019/volume-51-2015/issue-1/vol51_no1_2.pdf