

**ZMATH 2012a.00555****Langer, Joel C.; Singer, David A.****Reflections on the lemniscate of Bernoulli: the forty-eight faces of a mathematical gem.**

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This article provides an extensive study of Bernoulli's lemniscate and its symmetries as curve in complex projective space. It can be viewed as an immersed copy of the Riemann sphere on which the rotational octahedral group  $\mathcal{O}$  and the full octahedral group  $\hat{\mathcal{O}}$  act. The octahedral vertices correspond to the three complex double points, the edges are the fix-point curves of those antiholomorphic reflections that also fix four of the vertices. The  $\hat{\mathcal{O}}$ -orbit of the real lemniscate consists of six curves which triangulate the lemniscate as *tetrakis hexahedron*. Adding the octahedral edges produces a triangulation as *disdyakis dodecahedron*. The lemniscate can also be viewed as a Riemann surface with  $\mathcal{O}$ -invariant metric. Here, the disdyakis dodecahedron triangulation is produced by simple closed geodesics. The vertex orbits with respect to  $\hat{\mathcal{O}}$  have lengths six, eight, and twelve and correspond to critical points of the Gaussian curvature  $K$ . Short and concise proofs of these results are not the author's aim. They rather take the reader on a leisurely tour through many different fields such as complex mappings (inversion, stereographic projection, Joukowski maps, Schwarzian reflection), linkages, groups, Lagrangian subspaces, and quadrics. Mathematical truth and proofs evolve at a slow pace but leave a deep impression. The text is accompanied by attractive and informative pictures that nicely illustrate the lemniscate's symmetries in different models. Source code for *Mathematica* animations is given in an appendix.

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