ZMATH 2012e.00854

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Algebra and number theory. An integrated approach.

Hoboken, NJ: John Wiley & Sons (ISBN 978-0-470-49636-7/hbk). xi, 523 p. (2010).

Besides offering a compulsory one-semester course in elementary linear algebra (often compulsory also for many non-mathematics majors), an undergraduate program in mathematics in most, if not all, universities includes other compulsory one-semester courses in number theory, abstract algebra, and linear algebra. Usually, students take these three courses with different instructors, different textbooks, and in any order they like, often taking two, or even all three, of them in the same semester. Although students, and possibly instructors, find this fragmentation of algebra into three independent courses convenient, they realize that there are no well defined boundaries that separate these subjects, and especially the first two of them. The instructor of a course in number theory would possibly give a brief informal lecture on groups to show the students how the theorems of Fermat and Euler follow immediately from Lagrange's theorem. He or she would give a similar lecture on unique factorization in the rings $\mathbb{Z}\left[\sqrt{-1}\right]$ and $\mathbb{Z}[e^{2\pi i/3}]$ when discussing the Pythagorean equation $x^2 + y^2 = 1$ and the degree 3 Fermat equation $x^3 + y^3 = 1$. When it comes to the so-called Pell equation $x^2 - Ny^2 = 1$, the quadratic ring $\mathbb{Z}[\sqrt{N}]$ forces its way in. On the other hand, students without a background in number theory find difficulties when taking a course in abstract algebra, especially if they use a textbook like [*J. A. Gallian*, Contemporary abstract algebra. Boston, MA: Houghton Mifflin (2002; Zbl 1051.00001)], where number theory is everywhere. Thus even if these two courses are to be given in a predetermined order, it is not obvious which of them is to be a prerequisite of the other. To some extent, a similar case can be made about the interaction of the linear algebra course with these two courses. The authors of the book under review would like to view the subject matter of the three aforementioned courses as a single subject that may be called algebra and number theory, and that can be given in a sequence of three or four courses, where each course would be a blend of linear algebra, abstract algebra, and number theory. If such a viewpoint is adapted, then the book under review would be an ideal, and possibly a unique, textbook for such a sequence. The authors believe that 'this integrated approach will help build a deeper understanding of the subject in the students, as well as improve their retention of knowledge', and will also 'be helpful in using class time more efficiently'. This reviewer agrees that fragmentation of mathematics into separate seemingly independent and unrelated courses can be harmful, and he believes that students should be encouraged to conceive of mathematics as a unified entity. The integrated approach that the book preaches is likely to generate fruitful discussions and debates pertaining to how algebra should be presented. Educators, instructors, and committees responsible for revising and updating study plans should find the authors' views and their implementation worthy of consideration. The book is well-written and covers, with plenty of exercises, the material needed in the three aforementioned courses, albeit in a new order. As for typos and oversights, it contains, for a first edition, a tolerable amount (starting with an extra occurrence of the word 'Preface' on the first page of the preface). It has the unusual feature of not using any symbol (such as a Q.E.D. or a halmos) to signify the end of a proof, when some are demanding more symbols to signify other things (such as the end of a remark). This reviewer also feels that there is an excessive use of boldface letters throughout. While he agrees that words like 'Theorem', 'Proof', 'Definition' should be highlighted, he feels that highlighting 'GCD', 'deg', 'mod', etc. is unjustifiable. *Mowaffaq Hajja (Irbid)*

Classification: U25 H15 F65
Keywords: textbook; algebra; number theory