Rational sine, cosine and tangent of rational angles. (Seni, coseni e tangenti razionali di angoli razionali.)

Summary: The paper employs elementary tools that are available to high school students in order to prove that sine and cosine take algebraic integer values on rational multiples of $\pi$. This fact is used to show that the only rational, or quadratic irrational, values attained by sine and cosine on rational multiples of $\pi$ are:

$0, \pm \frac{1}{2}, \pm \frac{\sqrt{2}}{2}, \pm \frac{\sqrt{3}}{2}, \pm 1.$

Classification: G60

Keywords: trigonometry; sine; cosine; tangent