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Adding some perspective to de Moivre's theorem: visualising the n-th roots of unity.

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Summary: Traditionally, z is assumed to be a complex number and the roots are usually determined by using de Moivre's theorem adapted for fractional indices. The roots are represented in the Argand plane by points that lie equally pitched around a circle of unit radius. The n-th roots of unity always include the real number 1, and also include the real number -1 if n is even. The non-real n-th roots of unity always form complex conjugate pairs. This topic is taught to students studying a mathematics specialism as an application of de Moivre's theorem with the understanding that the roots occur in the complex domain. Meanwhile, in the Cartesian plane, a closely related topic deals with the solution of polynomials. The aim of this paper is to demonstrate visually the connection between the reduced polynomial $y = x^n - 1$ in the Cartesian plane and the resulting n-roots which invariably appear in the Argand plane. There is no contradiction here: the reader will find a three-dimensional surface representation of Equation (2) provides the full link between both the Cartesian and Argand planes, and illustrates not only the location of the roots in relation to the original equation but also shows why they occur with conjugate pairings. Examples will be provided for the cases $n = 3$, $n = 5$ and $n = 8$ which will be sufficient to illustrate the general pattern that emerges. The approach adopted here is a natural extension of the surface visualisation techniques first presented by the author [ibid. 26, No. 2, 6–20 (2012; ME 2013d.00569)] for quadratic equations. (ERIC)

Classification: H30 F50
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