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Algebra. A teaching and source book.

Cham: Springer (ISBN 978-3-319-19733-3/pbk; 978-3-319-19734-0/ebook). xxii, 539 p. (2015).

This book covers the fundamental concepts and results in what is generally labelled as 'abstract', 'higher' or 'modern' algebra at a graduate level. The material is selected and presented so as to cover the needs of any student in quest for an advanced degree in mathematics – to understand how to construct a rigorous proof and to have at hand the information specific to this area in mathematics. In a beginning chapter, the basic notational conventions used throughout the book are presented. In Chapter 2, the posets are introduced as the proper context, which is both natural and most effective, for discussing in a unified manner subjects as Galois connections, the modular law, the Jordan-Hölder theorem and dependence theories culminating with various dimension notions. Presentation of posets emphasizes the role of Zorn's lemma as a ubiquitous tool all over algebra. The next four chapters are devoted to properties of groups and techniques essential in their study. The key concepts are carefully explained and proofs for fundamental results such as Sylow's theorems, the Burnside transfer and fusion theorems are obtained by considering group actions (transitive, primitive and multiply transitive coprime actions, more precisely). Chapter 5 contains a discussion of derived series, upper and lower central series, solvability and nilpotence, ending with Schur-Zassenhaus theorem for finite groups. In Chapter 6, one encounters another key-feature of the approach adopted by the authors: the free group on a set is defined to be the automorphism group of a certain tree with labelled edge directions. This definition has the advantage that entails easy derivation of the fact that freeness is inherited by any subgroup of a free group, as well as of the universal property of free groups. Various constructions and elementary properties of rings are presented in Chapter 7, while basic module theory is exposed in Chapter 8. Besides Noetherian and Artinian modules, here one defines projective and injective modules. Classes of integral domains (Euclidean, unique factorization, Dedekind, to be specific) are described in Chapter 9. The classification theorem for finitely generated modules over a principal ideal domain is given in Chapter 10, along with applications – the rational canonical form of an endomorphism of a finite dimensional vector space and Jordan canonical form. A longer chapter is devoted to field theory. After introducing transcendental, algebraic, separable elements or field extensions, an exposition of Galois theory for finite field extensions is provided. Solvability of equations by radicals and the primitive element theorem can not be overlooked. In Chapter 12, the subject of semiprime rings is developed to the extent required for a proof of the Artin-Wedderburn theorem. The final chapter consists of a light introduction to category theory. After fixing the specific language, the notion of tensor product is presented with the goal of emphasizing its ubiquity in multilinear algebra. Additional material is contained in appendices to some chapters. Thus, the part devoted to groups ends with a mention of the classification of finite simple groups, and the arithmetic of quadratic domains is deferred to an appendix to Chapter 9. Chapter 10 has two appendices in which the highly-motivated student can find basic information on valued fields and a proof for Wedderburn's theorem, respectively. A lot of interesting results are found in numerous exercises accompanying all but the first chapter. For many of them the authors generously offer detailed hints which help the curious student to produce fully fledged proofs. In fact, such an approach is explicitly and forcefully advocated by authors on page vii: "Real learning is basically curiosity-driven self-learning." Selection and organization of such a rich material greatly benefited from the long teaching experience of both authors. It is therefore surprising the abundance of all sorts of errors encountered in the book. Most of them are innocuous yet disturbing typos accumulated at all stages of book production, as diverse as wrong page/section references (see pp. 339, 424, 478), promised and never-given article references (e.g., pp. 438 and 441), 'principle ideal domain', 'greatest common denominator', 'it's' instead of 'its', 'more 'course' instead of 'coarser'. Chasing all the errors would provide students with a substitute for the PokemonGo frenzy. The only mathematically significant assertions which need revision are on page 206 ('the zero polynomial ... possesses every possible degree in $\mathbb{N}$') and page 239, lines 4 and 11–15. Due to an impressive amount of material presented in a very articulated manner, this book is a valuable addition to the literature. The learning objectives are clearly stated in the preface and the authors made all efforts to facilitate the learning process as well as the teaching activity. One of the strong points of the book is the information shared with readers in footnotes offering a broader perspective on the topic. It suffices to quote the description of abstract algebra given on page 2: 'There is a common misunderstanding of this word "abstract" that mathematicians seem condemned to suffer. To many, "abstract" seems to mean "having no relation to the world – no applications". Unfortunately, this is the overwhelming view of politicians, pundits of education, and even many university administrators throughout the United States. One hears words like "ivory tower", "intellectuals on welfare", etc. On the contrary, these people have it just backwards. A concept is "abstract" precisely because it has *more than one* application – not that it hasn't *any* application. It is very important to realize that two things introduced in distant contexts are in fact the same structure and subject to the same abstract theorems.'

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Classification: H45
Keywords: monoid; poset; total ordering; zornification; closure operator; chain conditions; semilattice; Jordan-Hölder theorem; group; coset; group homomorphism; normal subgruop; permutation group; transitive group action; primitive transitive action; Sylow's theorems; Burnside transfer and fusion theorems; derived series; central series; Schur-Zassenhaus theorem; Cayley graph; free group; Brauer-Ree theorem; ring; monoid ring; integral domain; module; Noetherian/Artinian module; integral element; Hilbert basis theorem; exact sequence of modules; projective/injective module; divisibility; Euclidean domain; principal ideal domain; unique factorization domain; localization; Dedekind domain; ideal class group; invariant factors; Jordan canonical form; field; field extension; splitting field; Chevalley-Warning theorem; separability; normal extension; Galois theory; Galois group; solvability of equations by radicals; semiprime ring; socle; completely reducible ring; Artin-Wedderburn theorem; category; tensor product; multilinear map; symmetric/exterior algebra
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