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**Mathematical analysis of problems in the natural sciences. Translated from the 2008 Russian original by Gerald Gould.**

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The present work tries to illustrate the relation between "pure mathematics" and natural sciences such as hydrodynamics, thermodynamics, statistical physics and information theory. Above all, it attempts to unify three topics of analysis and physics, the dimensional analysis of physical quantities, functions of very large numbers of variables, and classical thermodynamics and contact geometry. To every topic mentioned, a special part of the book is devoted. The book also contains many historical remarks. The author's popular article "Mathematics as language and method" is printed in the Appendix. In part 1 of the work, first the basic elements of dimension theory are introduced. The postulate of the absoluteness of ratios is discussed, which suggests that under the change of the scale of a basic unit, all physical quantities of the same type (all areas, all forces etc.) change their values in the same proportion. On the basis of the postulate, a function of dimension and a formula for the dimension of a physical quantity in a given basis are derived. The fundamental theorem of dimensions of physical theories is discussed, which helps e.g. to introduce dimensionless quantities, or to select the system of dimensionally independent variables. The content, sense and capability of the fundamental theorem, as well as the pitfalls associated with it, are explained considering concrete applications as the rotational period of a body in a circular orbit, Kepler's third law and the degree exponent in Newton's law of universal gravitation, the period of oscillation of a heavy pendulum, the outflow of volume and mass in a waterfall, as well as the drag force for the motion of a ball in non-viscous and viscous media. At the end of part 1, also further applications to hydrodynamical systems and turbulence are dealt with. The Navier-Stokes equation is considered, comments of bifurcations in dynamical systems are given, and Kolmogorov's model for turbulence is explained. Part 2 on multidimensional geometry and functions of a very large number of variables starts with a less popular example, the digital sampling of a signal. The transmission of information along a communication channel is considered in detail. The sampling theorem, Kotel'nikovs formula (the basis of modern digital representation of a signal) is introduced. Further, other, even more fundamental, applications of multiparameter phenomena and spaces of large dimensions are briefly mentioned, such as the molecular theory of matter, the phase space in classical Hamiltonian mechanics, the Gibbs thermodynamic ensembles, and probability theory. The larger the dimension $n$ of a ball-formed space, the larger its part of the volume which is concentrated in a small neighbourhood of its boundary. Based on this "principle of concentration" (the geometric analogue of the law of large numbers), in the textbook the Gauß distribution of statistical theory and the Maxwell distribution of kinetic theory are introduced. It is discussed why the principle of concentration leads to nearly orthogonality of random vectors in spaces with large $n$, and to almost constancy of functions on a sphere of very large dimension. Remarks are made on standard averages of functions and its median value, on $n$-dimensional cubes, thermodynamics and limiting distributions. Finally, in the second part, the discussions are supplemented with Shannon's theorem on the speed of transmission along a communication channel in the presence of noise. Part 3 of the work is dedicated to classical thermodynamics and contact geometry. Thus it starts with the explanation of the first and second laws of thermodynamics, the introduction of the terms energy and entropy, and the related mathematical setting. Adiabatic transitions occurring without heat exchange and Carathéodody's axiom (in any neighborhood of an equilibrium thermodynamic state of a system there are equilibrium states into which it is impossible to pass by an adiabatic process) are briefly reviewed. Further, also a full mathematical formulation of the axiom is given, and the problems and after-effects it leads to are analysed. In mathematics the axiom means that in the neighbourhood of any point of a space of states there are points that cannot be accessed by an admissible path from a given point. The mathematics leads to the application of contact geometry to the main principles of thermodynamics in the language of differential forms, contact distributions, the Frobenius theorem and the Carnot-Carathéodory metric. The final chapter of the work deals with classical and statistical thermodynamics. The Gibbs ensemble and the "thermodynamicization" of mechanics are described. Paradoxes, problems and difficulties are discussed. A few words concerning the quantum statistical thermodynamics are told. In the first instance, the textbook is intended for mathematicians, but it may also be useful to students and specialists of natural sciences.

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*Keywords:* mathematical analysis; thermodynamics; dimensional analysis; contact geometry; thermodynamics; statistical physics; information theory; functions of large numbers of variables; law of large numbers; Gauss distribution; Maxwell distribution; principle of concentration; median value; limiting laws of ther-

modynamics; n-dimensional cubes; Shannon's theorem; energy; entropy; Carathéodory's axiom; Frobenius theorem; Carnot-Carathéodory metric; Gibbs ensemble quantum statistical thermodynamics; classical thermodynamics; kinetic theory
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