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Teaching the calculus.

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Summary: Methods of teaching the Calculus are presented in honour of Sir Isaac Newton, by discussing an extension of his original proofs and discoveries. The methods, requested by Newton to be used that reflect the historical sequence of the discovered Fundamental Theorems, allow first-time students to grasp quickly the basics of the Calculus from its original foundation. It is necessary to present the information along with other methods one might employ now in common use; avoiding these principles of the Calculus can cloud its simplicity. The nature and significance of integration and differentiation are clarified, including the power and chain rules, logarithmic differentiation, and the fundamental relationships between integrals and derivatives. By considering in detail the graphical areas covered by a thin thread under a continuous smooth function along its course, the Fundamental Theorems of the Calculus may be proven. Any function $f(x)$ is both the derivative of and the integrand of its integral $F(x)$. The area traced out by any $f(x)$ has ordinal values that are the slopes of $F(x)$ because the function is the derivative of the integral. Differences in ordinal values of an integral, $F(b) - F(a)$, calculate the total of all dy variations along the integral and equal the exact net area of thin vertical $f(x)$dx threads between the derivative function and the horizon from a to b. Unique features of the sine function, including its line integral arc length and its area, demonstrate the power of the Fundamental Theorems of the Calculus.

Classification: I40 I50
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