

ZMATH 2012e.00492

Tall, David; Yevdokimov, Oleksiy; Koichu, Boris; Whiteley, Walter; Kondratieva, Margo; Cheng, Ying-Hao

Cognitive development of proof.

Hanna, Gila (ed.) et al., Proof and proving in mathematics education. The 19th ICMI study. Berlin: Springer (ISBN 978-94-007-2128-9/hbk; 978-94-007-2129-6/ebook). New ICMI Study Series 15, 13-49 (2012).

Summary: This article traces the long-term cognitive development of mathematical proof from the young child to the frontiers of research. It uses a framework building from perception and action, through proof by embodied actions and classifications, geometric proof and operational proof in arithmetic and algebra, to the formal set-theoretic definition and formal deduction. In each context, proof develops over the long-term from the recognition and description of observed properties and the links between them, the selection of specific properties that can be used as definitions from which other properties may be deduced, to the construction of ‘crystalline concepts’ whose properties are a consequence of the context. These include Platonic objects in geometry, symbols having relationships in arithmetic and algebra and formal axiomatic systems whose properties are determined by their definitions.

Classification: E50 C30 D30

Keywords: mathematical proof; cognitive development; perception and action; geometric proof; operational proof; formal set-theoretic definition; formal deduction

doi:10.1007/978-94-007-2129-6_2