ZMATH 2012f.01035

Whiteley, Walter J.; Mamolo, Ami

The popcorn box activity and reasoning about optimization.

Math. Teach. (Reston) 105, No. 6, 420-426 (2012).

Summary: A well-known optimization problem is the Popcorn Box investigation, which involves a movie theater snack container. The problem has been tailored for classroom investigations by the Ontario Association for Mathematics Education. The exploration was designed for students in grades 9 through 12. A common strategy proposed for algebra students is to make a sequence of square cuts of varying sizes and compare the calculated volumes in a table of values. Students can then estimate when the maximum volume is achieved. The typical conjecture – that the optimum volume occurs when the box is a cube – is not correct, a result that is surprising to many students. This iterative approach through approximation and repeated application of the volume formula to different box sizes allows algebra students to see that the volume changes–one of the big ideas of the investigation – and that one (and perhaps several) of the samples has a biggest volume. However, this approach offers little insight into why the optimum occurs where it does. In this article, the authors present a new approach to this classic problem, one that stimulates students' geometric reasoning using three-dimensional models. This new approach also sheds light on what is considered a key idea in optimization – why the optimum occurs where it does. The activity offers a chance to explore key concepts specific to geometric optimization – in particular, that the rate of change of volume is represented by surface areas and that optimizing volume can be seen as a balance of the loss and gain of the associated areas. Further, the activity invites students to explore and track rates of change visually and geometrically. The design of this activity affords ways to direct and shift students' attention and to discuss these attention shifts in a collective way. Such shifts have been recognized as important aspects of mathematical thinking, although they may often be hidden in the mind of the learner. The authors conclude this article with comments on the pedagogical implications and limitations of their approach to the popcorn box optimization problem. (ERIC)

Classification: N60 M90
Keywords: algebra; geometric concepts; mathematics instruction; problem solving; computation; learning activities; manipulative materials; optimization
doi:10.5951/mathteacher.105.6.0420 http://www.nctm.org/publications/article.aspx?id=32047