ZMATH 2013a.00873

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Numerical methods for eigenvalue problems.

De Gruyter Graduate Lecture. Berlin: De Gruyter (ISBN 978-3-11-025033-6/pbk; 978-3-11-025037-4/ebook). viii, 208 p. (2012).

The texbook presents the most important numerical methods for the finding of eigenvalues and eigenvectors of matrices. Some sections are devoted to generalized eigenvalue problems for polynomials by spectral parameter matrices. The texbook is intended as the ground for a short course for third- or fourth-year undergraduate students. The introductory Chapter 1 contains some examples of eigenvalue problems arising in structural mechanics, stochastic processes and systems of ordinary differential equations. Chapter 1 gives auxiliary tools such as characteristic polynomials, properties of Hilbert spaces, invariant subspaces, Schur decomposition, similarity and non-unitary transformations. In Chapter 3, the Jacobi iterations are considered allowing to reduce a given $(n \times n)$-selfadjoint matrix to diagonal form. Not being the fastened method it is guaranteed to converge for any selfadjoint matrix. The separate Sec. 3.5 presents the Jacobi iterations converging quadratically for matrices close to diagonal. Chapter 4 contains the power Rayleigh and simultaneous iterations as simple methods for eigenvectors computation with accuracy estimations for approximations to eigenvalues and eigenvectors and stopping criteria for iterations. In Chapter 5, algorithms for the finding of all eigenvalues and eigenvectors at the usage of QR iterations having quadratic and even cubic convergence for their computations. In Chapter 6, the bisection methods are presented applicable usually for eigenvalues computations for selfadjoint matrices reduced to tridiagonal form after Householder transformations, guaranteeing the convergence with 1/2 rate. Here, the Gershgorin circles are used to obtain a reliable initial guess for the bisection algorithm. Chapter 7 introduces Krylov subspace methods for large sparse eigenvalue problems on the base of successive steps: projection methods introduction for large sparse eigenvalue problems; Krylov subspaces construction with their basic properties investigation; the Arnoldi iteration and the symmetric Lanczos algorithm introduction as examples of Krylov subspace methods; the Chebyshev polynomials usage to analyze the convergence behaviour of Krylov subspace methods. In Chapter 8, generalized eigenvalue problems are investigated at the usage of the QZ algorithm based on the generalization of the Schur decomposition. It contains also some results for polynomial eigenvalue problems which are reduced here to generalized eigenvalue problems at the usage of a linearization. Every chapter is equipped with several exercises and summaries to help the beginners with the best understanding of the relevant material.

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Classification: N15 N45

Keywords: eigenvalue problems; linear operators in finite-dimensional spaces; large sparse matrices; textbook; eigenvectors; characteristic polynomials; invariant subspaces; Schur decomposition; Jacobi iteration; algorithm; QR iteration; convergence; bisection methods; Householder transfromation; Gershgorin circles; Krylov subspace methods; Arnoldi iteration; Lanczos algorithm; QZ algorithm; polynomial eigenvalue problems; generalized eigenvalue problem

doi:10.1515/9783110250374