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Underwood, Robert G.
Fundamentals of modern algebra. A global perspective.
Hackensack, NJ: World Scientific (ISBN 978-981-4730-28-0/hbk). x, 220 p. (2016).

This textbook contains a concise introduction of three fundamental algebraic structures: groups, rings, modules. The material is sufficiently rich to allow for an exposition of basic algebraic number theory, along with rudiments of the theory of finite fields. The book consists of five chapters, the first of which is devoted to groups. The reader first learns basic notions (subgroup, normal subgroup, cosets, quotient group, homomorphisms) and results (Lagrange's theorem, the three isomorphism theorems, universal property for kernels). Then the author proceeds to the structure theorem for finitely generated abelian groups, Cauchy's theorem, and Sylow's theorems. The major theme of Chapter 2 (entitled "Rings") is the construction of rings of matrices, polynomials or fractions. A swift discussion of arithmetic properties of commutative rings includes results valid in the ring of univariate polynomials over a field, principal ideal domains, unique factorization domains or completions of the field of rational numbers with respect to an absolute value. The notion of Noetherian ring, along with Hilbert's basis theorem, are also introduced. The concept of modules over a commutative ring with unity is introduced in Chapter 3. Several properties of vector spaces are extended to modules over principal ideal domains or Noetherian rings. In this context, fundamental results on free and projective modules are discussed. The reader also encounters here bilinear maps, the tensor product of two modules, commutative algebras, the dual of free modules with respect to a bilinear form. The chapter ends with a study of the discriminant of modules over an integral domain. The knowledge conveyed in the first part of the book allows the author to proceed with an exposition of the foundations of algebraic number theory. This is achieved in Chapter 4, which starts with a study of simple algebraic field extensions, detailed for a subfield of the complex numbers field. In this framework, Galois groups are defined and the fundamental theorem of Galois theory is proven. It is shown that the ring of integers is a Dedekind domain, that unique factorization for ideals holds, and a construction for its class group is indicated. The groups of units of rings of integers are explicitly determined in the case of cyclotomic fields and of quadratic extensions of $\mathbb{Q}$. The final section of Chapter 4 is concerned with extensions of the field of p-adic rationals. Various questions on finite fields are studied in Chapter 5, including existence and uniqueness up to isomorphisms, the relationship between the order of an irreducible polynomial and the order of any of its roots in a relevant group of units. Some of the results given here are used in the study of linearly recurrent sequences over finite fields. Each chapter ends with several dozens of exercises of varying degree of difficulty. None of the answers/solutions is provided, but in many places the author has worked out the details of illuminating examples. The account of so much material is necessarily concise, with the most difficult or longest proofs replaced by pointers to an accessible place in the literature where to be found. However, such a short and to the point exposition has a downside – it is prone to give rise to misunderstandings (e.g., from p. 25 one learns that any two cyclic groups are isomorphic; the definition for the degree of a polynomial given on p. 44 lacks precision) and omissions (e.g., the credit for the solution to the problem of the tenth complex quadratic field with class-number one). The book assumes no background except familiarity with linear algebra. Its target audience consists of first-year graduate students (level L in those universities which adopted the LMD classification scheme). *Mihai Cipu (Bucureşti)*

Classification: H45
Keywords: group; subgroup; normal subgroup; cosets; quotient group; homomorphism; Lagrange's theorem; universal property for kernels; structure theorem for finitely generated abelian groups; Cauchy's theorem; Sylow's theorems; ring of matrices; polynomial ring; ring of fractions; principal ideal domains; unique factorization domains; absolute value; Noetherian ring; Hilbert basis theorem; free modules; projective modules; bilinear maps; tensor product of modules; commutative algebras; dual of free modules with respect to a bilinear form; discriminant of module; simple algebraic field extensions; Galois groups; fundamental theorem of Galois theory; Dedekind domain; class group; cyclotomic fields; finite fields; linearly recurrent sequences
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