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Proof of the continuity at a point for a class of functions of two independent variables by construction.

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From the text: When a new mathematical topic is to be presented in the classroom, examples that demonstrate the significant related concepts should be included. However, examples for some topics are deficient, lengthy, or nonexistent. Consider a lesson for a multivariable calculus class on the continuity of a function of two independent variables at a point of its domain. A unique limit of the function at a point of the domain is necessary. Examples presented in many texts primarily demonstrate the process of showing when a limit does not exist. For these types of examples, one simply shows the existence of two different limits of a function that are obtained along two different paths to the given point in the domain, respectively. A proof establishing the existence of a unique limit along all possible paths to that given domain point is sometimes briefly described. Thus some students are probably not directly experiencing the derivation of the existence of a unique limit. This article presents a direct proof that establishes the continuity of any multi-variable polynomial function at any given domain point. It is a method that constructs a mathematical statement that is used to prove the existence of a unique limit. Students can use this method to prove continuity at a point, and thus experience, in a positive sense, the manipulation of the mathematical structure of an epsilon-delta argument. This article may be of interest to instructors and students of multivariable calculus.

Classification: I65 I25 E55
Keywords: multivariable calculus; polynomial functions; continuity at a point; proofs; circles; limits; heuristics; concept formation; approach