ZMATH 2016c.00971
Gordon, Sheldon P.; Yang, Yajun
Deriving Simpson's rule using Newton interpolation.
Math. Comput. Educ. 50, No. 1, 34-41 (2016).

From the text: Almost every numerical integration technique is based on the idea of fitting a series of polynomials to a function using successive subsets of (usually horizontally uniformly spaced) points, approximating the area under each portion of the graph of the function with the area under the corresponding polynomial, and summing the results. For instance, Simpson's rule fits one quadratic function to the first group of three points, then another quadratic to the next group of three points, and so forth. Simpson's rule is typically derived in one of two ways, either by using the Lagrange interpolation formula (particularly in numerical analysis courses) or, more frequently (in calculus courses), by writing the quadratic in the form $Q(x) = A + B(x - x_0) + C(x - x_0)^2$ for the first polynomial and then setting up a system of linear equations in the three unknowns based on the three points to solve for A, B, and C. Some relatively simple algebra lets one solve this system of equations for the three coefficients and then a simple integration of the resulting quadratic function from $x = x_0$ to $x = x_2$ gives an approximation to the area under the original curve from $x = x_0$ to$x = x_2$. When repeated over successive sets of points, Simpson's rule results. We now look at an alternative approach to deriving Simpson's rule, one that can be easily extended to derive numerical integration methods of higher degree and that seems to be more direct, more in the spirit of a calculus course, and perhaps more elegant. It is based on Newton's forward difference interpolating formula.

Classification: N40 N50
Keywords: numerical analysis; numerical integration; Simpson's rule; Newton interpolation polynomial; approximation; quadratic polynomials; area under a curve; error functions; intermediate value theorem

 October 14, 2019