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Linear algebra and geometry. Translated from the Russian by David Kramer and Lena Nekludova.

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Igor R. Shafarevich (born 1923), one of the towering figures in Russian mathematics of the 20th century, is a renowned researcher and teacher in algebra, number theory, and algebraic geometry. Apart from his pioneering research contributions in these fields, he is particularly well-known through his numerous popular textbooks in these areas of mathematics. The book under review grew out of I. R. Shafarevich's courses on linear algebra and geometry, which he taught from the 1950s through the 1970s at the Faculty of Mechanics and Mathematics of Moscow State University. As pointed out in the preface, the notes for some of these lectures were preserved in the faculty library, and these were used in preparing the present book. Also, the authors have included some topics that were discussed in student seminars at the time, and they have enriched the material by various additional items and examples for further illustration. Those are marked with an asterisk for specific characterization and possible omission. As for the precise contents, the book consists of fourteen chapters, each of which is subdivided into several sections. After some preliminaries concerning sets, mappings, and basic topological notions for metric spaces, Chapter 1 starts off with the study of systems of linear equations by means of the Gaussian elimination procedure. Chapter 2 treats matrices and determinants, while Chapter 3 introduces vector spaces, linear transformations, dual vector spaces, and polynomial functions on vector spaces. Chapter 4 discusses endomorphisms of vector spaces, eigenvectors and invariant subspaces, the complexification of a real vector space, and orientations of real spaces. Chapter 5 explains the Jordan normal form of an endomorphism of a complex vector space, with applications to real square matrices and to systems of ordinary differential equations. Chapter 6 turns to quadratic and bilinear forms, with special emphasis on the concept of orthogonality and normal forms. The structure theory of symmetric and Hermitian forms is presented at the end of this chapter. Euclidean spaces are the central topic of Chapter 7, where also orthogonal transformations, symmetric transformations and orientations are analyzed in detail. Applications to mechanics and differential geometry as well as a discussion of pseudo-Euclidean spaces and Lorentz transformations complete the content of this chapter. Chapter 8 is devoted to affine spaces and affine transformations, with motions in affine Euclidean spaces appearing as a special topic. A first introduction to projective analytic geometry is provided in Chapter 9, including projective spaces, projective transformations, the notion of cross ratio, and topological properties of projective spaces. Chapter 10 explains exterior products of vectors and exterior algebras which is done here in the geometric context of Plücker coordinates, Plücker relations, and Grassmannians. In an appendix, the connection between exterior algebra and determinantal calculus is expounded. Chapter 11 gives a very detailed analysis of quadrics in various types of spaces, whereas Chapter 12 discusses the rudiments of hyperbolic (or Lobachevskian) geometry, and that both analytically and synthetically. Apart from some instructive formulas in hyperbolic geometry, this chapter also depicts the historical development of this topic. Chapter 13 leaves the realm of linear algebra and geometry in that it turns to groups, rings, and modules. The focus is here on the structure theorem for finite abelian groups, on the one hand, and that for finitely generated torsion modules over a Euclidean ring on the other. The aim of this chapter is to demonstrate the use of some more abstract algebraic concepts, especially in analogy with the Jordan normal form for endomorphisms of vector spaces. Finally, Chapter 14 develops the elements of the theory of finite-dimensional representations of finite groups, in a way as a connecting link between group theory and linear algebra. This includes the most basic concepts of representation theory in general, which is then followed by a specific description of representations of finite groups, irreducible representations, and irreducible complex representations of abelian groups via characters, respectively. At the end of the book, the authors present a brief chronology of the appearance of the various concepts discussed in the course of the text. The bibliography mainly refers to those books that belonged to the standard textbook literature at the time when I. R. Shafarevich's original lectures were given, including many classical Russian texts on linear algebra and geometry. All in all, this beautiful textbook not only reflects I. R. Shafarevich's unrivalled mastery of mathematical teaching and expository writing, but also the didactic principles of the Russian mathematical school in teaching basic courses such as linear algebra and analytic geometry. One has to recall that generations of brilliant Russian mathematicians have been introduced to these areas of mathematics in this particular way, which is characterized by a sound balance of concreteness and abstraction, intuitive perception and formalism, as well as theory and applications. It is more than gratifying that I. R. Shafarevich's lectures, from more than half a century ago, have been made accessible to a wide audience of international readers, and to further generations of students, too. Studying this textbook means learning from one of the great masters in contemporary mathematics, on the one hand, and to see that linear algebra and geometry once were taught as an inseparable unit. In addition, this book may be regarded as a historical document in the relevant textbook literature, just as a part of the mathematical heritage of the great mathematician I. R. Shafarevich, who will celebrate his honourable ninetieth birthday in a few months. *Werner Kleinert (Berlin)*

Classification: H65 G75
Keywords: linear algebra (textbook); vector spaces; linear transformations; matrices; bilinear forms; affine spaces; hyperbolic spaces; Euclidean spaces; hyperbolic geometry; representations; systems of linear equations; Gaussian elimination; determinants; eigenvalues; invariant subspace; Jordan normal form; Hermitian form; orthogonal transformation; symmetric transformation; Lorentz transformation; affine transformation; projective analytic geometry; projective transformation; projective space; exterior products; exterior algebras; Plücker coordinates; Plücker relations; Grassmannians; Lobachevskian geometry; finite abelian groups; torsion modul; Euclidean rings; finite groups; irreducible representations; abelian groups
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